Contact mechanics and elements of tribology

Lecture 7.

Two words about friction

Vladislav A. Yastrebov

MINES ParisTech, PSL Research University, Centre des Matériaux, CNRS UMR 7633, Evry, France

@ Centre des Matériaux
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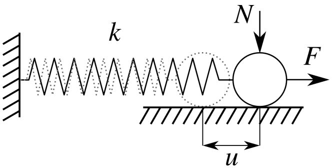
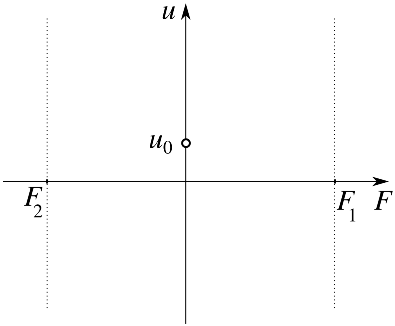
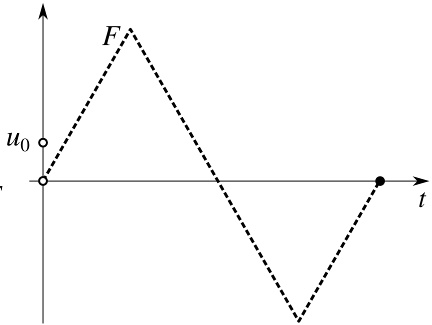
Coulomb's friction

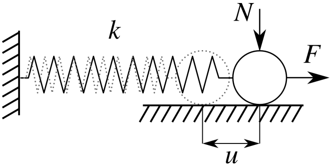


One-dof frictional problem

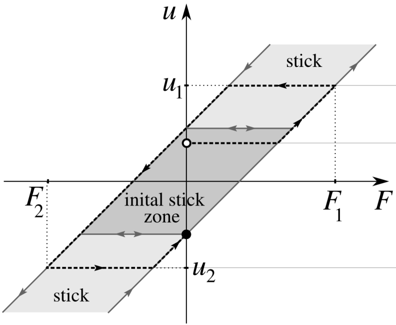


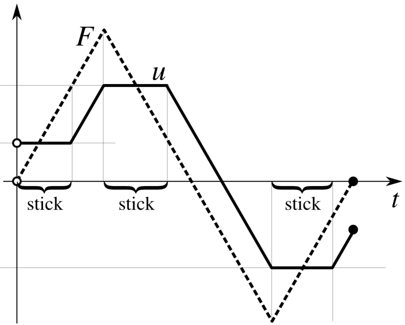


Coulomb's friction



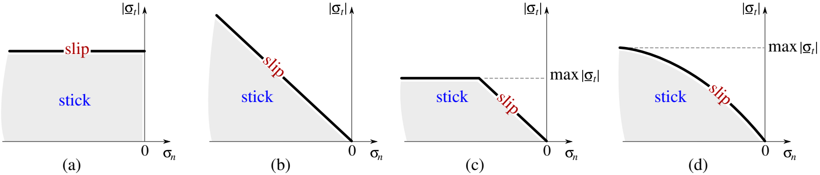
One-dof frictional problem





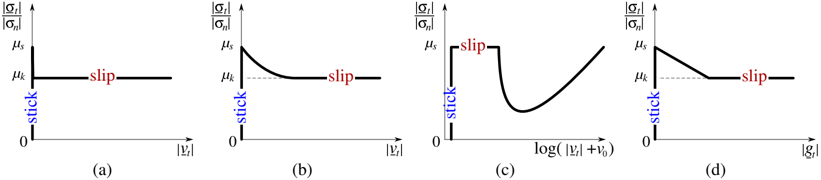
More friction laws

- **Static criteria**



(a) Tresca (b) Amontons-Coulomb (c) Coulomb-Orowan (d) Shaw

- **Kinetic criteria**



(a,b) velocity weakening (c) velocity weakening-strengthening
(d) Linear slip weakening

- μ_s static and μ_k kinetic coefficients of friction.

Theory and simulations

- ⋆ **Theory**

1. M. Renardy, Ill-posedness at the boundary for elastic solids sliding under Coulomb friction, J. Elast. 27:281-287 (1992).
2. G. G. Adams, Self-excited oscilations of two elastic half-spaces sliding with a constant coefficient of friction, J. Appl. Mech. 62:867-872 (1995).
3. J. A. C. Martins, F. M. F. Simões, On some sources of instability/ill-posedness in elasticity problems with Coulomb's friction, in Contact Mechanics, eds. M. Raous, M. Jean, J.-J. Moreau, 95-106 (1995).

- ⋆ **Simulation**

1. D. J. Andrews, Y. Ben-Zion, Wrinkle-like slip pulse on a fault between different materials, J. Geophys. Res., 102:553-571 (1997).
2. R. A. Harris, S. M. Day, Effect of a low-velocity zone on a dynamic rupture, Bull. Seismol. Soc. Am., 87:1267-1280 (1997).
3. A. Cochard, J. R. Rice, Fault rupture between dissimilar materials: ill-posedness, regularization, and slip-pulse response, J. Geophys. Res. 105:25891 (2000).

- ⋆ **Theory**

1. K. Ranjith, J. R. Rice, Slip dynamics at an interface between dissimilar materials. J. Mech. Phys. Solids 49:341-361 (2001).

Rate and state friction and regularization

Rate and state friction law

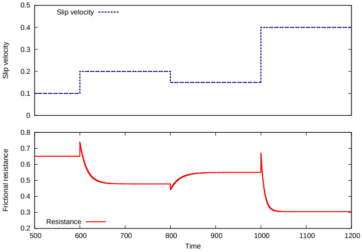
- Rate $v_t = |\underline{v}_t|$ – relative slip velocity
- State θ – ≈ internal time
- Dieterich–Ruina–Perrin (1979, 83, 95)

  Frictional resistance

  $\sigma_t^c = |\sigma_n| \left[f_s + b\theta + a \ln(v_t/v_0)\right]$

  Evolution of the state variable

  $\dot{\theta} = -\frac{v_t}{L}\left[\theta + \ln\left(\frac{v_t}{v_0}\right)\right]$

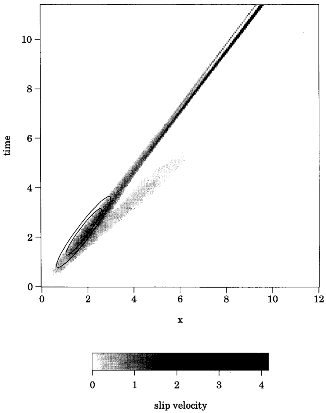


Rate and state friction law

One story

- Spliting of slip pulses ☺

  [1] D. J. Andrews, Y. Ben-Zion, Wrinkle-like slip pulse on a fault between different materials, J. Geophys. Res., 102:553-571 (1997)



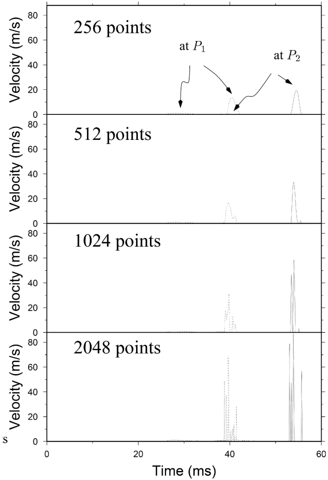
Slip pulse splitting[1]

One story

- Spliting of slip pulses ☺

  [1] D. J. Andrews, Y. Ben-Zion, Wrinkle-like slip pulse on a fault between different materials, J. Geophys. Res., 102:553-571 (1997)

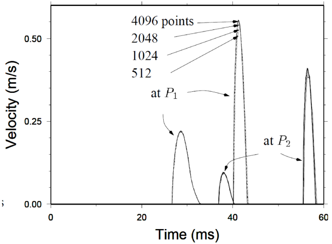
- Problems with mesh convergence



Mesh convergence problems[2]

One story

- Spliting of slip pulses ☺
  [1] D. J. Andrews, Y. Ben-Zion, Wrinkle-like slip pulse on a fault between different materials, J. Geophys. Res., 102:553-571 (1997)
- Problems with mesh convergence
- Regularized Prakash-Clifton law
  $\dot{\sigma}_t = -\frac{v_t}{L}(\sigma_t + \mu\sigma_n)$
  [2] A. Cochard, J. R. Rice, Fault rupture between dissimilar materials: ill-posedness, regularization, and slip-pulse response, J. Geophys. Res. 105:25891 (2000)
- Theory
  [3] K. Ranjith, J. R. Rice, Slip dynamics at an interface between dissimilar materials. J. Mech. Phys. Solids 49:341-361 (2001)



Using Prakash-Clifton friction law regularizes the problem[2]

		20% contrast; c_{GR} defined		30% contrast; c_{GR} not defined		
	f	0	$f_{crit} \approx 0.22$ … $+\infty$	0	$f_{crit_1} \approx 0.15$	$f_{crit_2} \approx 1.75$ … $+\infty$
Coulomb law	Modal analysis	Unstable modes, R independent of k $V_{prop} \approx +c_{GR}$	$V_{prop} \approx +c_{GR}$ and/or $\lvert V_{prop}\rvert \gtrapprox P_{slow}$ ($V_{prop} < 0$)	Stable modes	Unstable modes, R independent of k $V_{prop} \approx +S_{slow}$	$V_{prop} \approx +S_{slow}$ and/or $\lvert V_{prop}\rvert \gtrapprox P_{slow}$ ($V_{prop} < 0$)
	Realistic cases	Ill posed: no convergence through grid size reduction		Well posed; only dying pulses	Ill posed: no convergence through grid size reduction	
Modified Prakash-Clifton law	Modal analysis	Unstable modes V_{prop} (weak) function of k R function of k		Stable modes	Unstable modes V_{prop} (weak) function of k for $k < k_{crit1}$	for $k < k_{crit2}$ ($k_{crit2} > k_{crit1}$)
	Realistic cases	Regularization of ill-posedness at large k Self-sustained pulses may propagate at $V_r \lesssim +c_{GR}$	$V_r \lesssim +c_{GR}$ and/or $V_r \approx -P_{slow}$	Well posed; only dying pulses	Regularization of ill-posedness Self-sustained pulses may propagate at $V_r \approx +S_{slow}$	$V_r \approx +S_{slow}$ and/or $V_r \approx -P_{slow}$

Table for two combinations of materials[2]

Introduction: Regularized friction law

Elastodynamic frictional contact problem may be ill-posed/unstable

- For two identical materials: no problems
- Two different materials:
  - If generalized Rayleigh speed is defined: unstable
  - If generalized Rayleigh speed is not-defined: stable for $f < f_c$
    Critical friction exists.

Introduction: Regularized friction law

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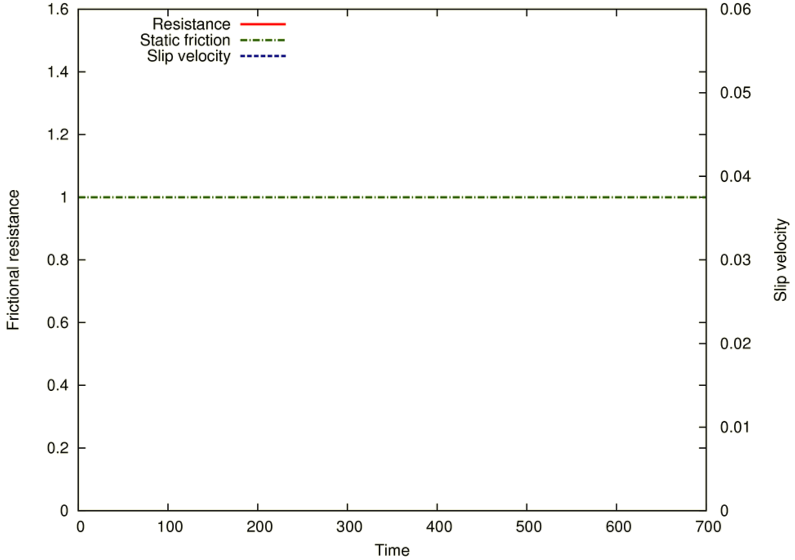
Possible solutions

- Abandon the proportionality of frictional strength to contact pressure
  $|F_t| \neq \mu |F_n|$
- Viscous analogy
- Experiment based Prakash-Clifton friction law [1]
  $\dot{F}_t = -\frac{|V|+V_0}{L} \left[F_t - \mu \max(0, |F_n|) \right]$
- Non-local friction [2]
  $|F_t(x)| = \mu \frac{1}{2L} \int\limits_{-L}^{L} |F_n(x-s)| w(x-s) ds$

[1] Prakash, Clifton. Time resolved dynamic friction measurements in pressure-shear. In Experimental Techniques in the Dynamics of Deformable Solids, edited by Ramesh, vol. 165:33-48, ASME, New York (1993).

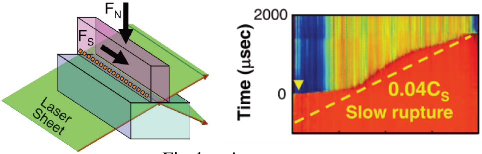
[2] J. A. C. Martins, F. M. F. Simões, On some sources of instability/ill-posedness in elasticity problems with Coulomb's friction, in Contact Mechanics, eds. M. Raous, M. Jean, J.-J. Moreau, 95-106 (1995).

Introduction: Regularized friction law

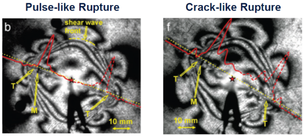


Introduction: lab experiments

1. K. Xia, **A. J. Rosakis**, and H. Kanamori, Science 303, 1859 (2004).
2. S. M. Rubinstein, G. Cohen, and **J. Fineberg**, Nature 430, 1005 (2004).
3. S. M. Rubinstein, G. Cohen, and **J. Fineberg**, Phys. Rev. Lett. 98, 226103 (2007).
4. O. Ben-David, G. Cohen, and **J. Fineberg**, Science 330, 211 (2010).
5. S. Latour, T. Gallot, S. Catheline, C. Voisin, F. Renard, E. Larose, and M. Campillo, Europhys. Lett. 96, 59003 (2011).



Fineberg's group



Rosakis' group

Conclusion

- It was an overview
- It is up to you to dig and understand the details

  "The devil is in the detail" or *"God is in the detail"*

- Good luck to you & Bon courage !