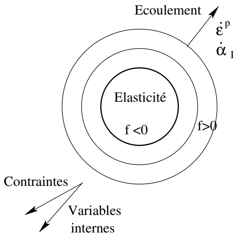

DMS 2015 MODELES A CRITERES MULTIPLES

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Plan

1. Introductive remarks
2. Ingredient of classical elasto-(visco)plastic constitutive equations
3. J_2 multimechanism models
   - Formulation
   - 2M2C and 2M1C
   - Typical behavior of 2M2C and 2M1C models
   - Ratchetting for 2M2C and 2M1C
4. Identification of the model
5. Concluding remarks
6. Bonus : extensions of the model

Purpose of the talk

- Briefly present the thermodynamical framework for unified approaches
- Introduce multimechanism models, J_2 type
- Illustrate the capabilities of the new model with reference to a ratchetting data base on 316 Stainless Steel

State variable, hardening variables

The free energy Ψ, used as a potential, defines stress and hardening variables knowing elastic strain and state variables

- Reversible part of the model
  - Elastic strain $\underset{\sim}{\varepsilon}^e$ and stress $\underset{\sim}{\sigma}$

$$\underset{\sim}{\sigma} = \rho \frac{\partial \Psi}{\partial \underset{\sim}{\varepsilon}^e}$$

- Dissipative part of the model
  - Strain like variables : *State variables* α_I
  - Stress like variables : *Hardening variables* A_I

$$A_I = \rho \frac{\partial \Psi}{\partial \alpha_I}$$

Classical isotropic + kinematic hardening

$$\Psi = \Psi^e + \Psi^p$$

- State variables $\underset{\sim}{\varepsilon}^e$, $\underset{\sim}{\alpha}$, r
- Stress $\underset{\sim}{\sigma}$
- Hardening variables $\underset{\sim}{X}$ and R

$$\Psi^e = \frac{1}{2}\underset{\sim}{\varepsilon}^e : \underset{\approx}{\Lambda} : \underset{\sim}{\varepsilon}^e \qquad \underset{\sim}{\sigma} = \underset{\approx}{\Lambda} : \underset{\sim}{\varepsilon}^e$$

$$\Psi^p = \frac{1}{3}C\underset{\sim}{\alpha} : \underset{\sim}{\alpha} + \frac{1}{2}bQR^2 \qquad \underset{\sim}{X} = \frac{2}{3}C\underset{\sim}{\alpha}$$

$$R = bQr$$

Elastic part is then fully characterized ($\underset{\sim}{\varepsilon}^e$ is observable)

Dissipative part

- Yield function $f(\underset{\sim}{\sigma}, \underset{\sim}{X}, R)$
- Flow potential $\Omega(f)$ + normality rule

$$\dot{\underset{\sim}{\varepsilon}}^p = \frac{\partial \Omega}{\partial \underset{\sim}{\sigma}} = \frac{\partial \Omega}{\partial f}\frac{\partial f}{\partial \underset{\sim}{\sigma}} = \dot{p}\underset{\sim}{n}$$

- Hardening potential $\Omega_h(\underset{\sim}{\sigma}, \underset{\sim}{X}, R)$

$$\dot{\underset{\sim}{\alpha}} = -\frac{\partial \Omega_h}{\partial \underset{\sim}{X}} \qquad \dot{r} = -\frac{\partial \Omega_h}{\partial R}$$

- Example 1 : linear hardening, $\underset{\sim}{\alpha} = \underset{\sim}{\varepsilon}^p \quad r = p$

$$f(\underset{\sim}{\sigma}, \underset{\sim}{X}, R) = J(\underset{\sim}{\sigma} - \underset{\sim}{X}) - R - \sigma_y \qquad \Omega = \Omega_h = \frac{n+1}{K}\left\langle \frac{f}{K} \right\rangle^{n+1}$$

$$\textit{Notation :} \qquad \dot{p} = \frac{\partial \Omega}{\partial f} = \left\langle \frac{f}{K} \right\rangle^{n} \qquad \underset{\sim}{n} = \frac{\partial f}{\partial \underset{\sim}{\sigma}}$$

$$\dot{\alpha} = \dot{p}\underset{\sim}{n} \qquad \dot{r} = \dot{p}$$

Generalized normality rule *if* $\Omega_h \equiv \Omega$

Nonlinear hardening

- Additional terms in the potential

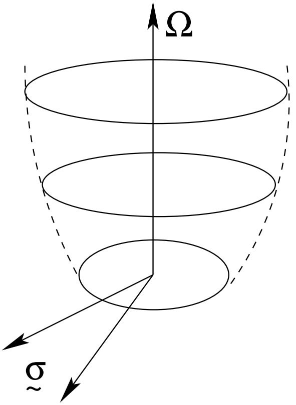
$$\Omega = \frac{n+1}{K}\left\langle \frac{f}{K} \right\rangle^{n+1} \qquad \Omega_h = \Omega + \frac{3D}{4C} \underset{\sim}{X} : \underset{\sim}{X} + \frac{R^2}{2Q}$$

$$\underset{\sim}{\dot{\alpha}} = \underset{\sim}{\dot{\varepsilon}}^p - \frac{3D}{2C} \underset{\sim}{X} \dot{p} \qquad \dot{r} = (1 - \frac{R}{Q})\dot{p}$$

- Hardening variables

$$\underset{\sim}{\dot{X}} = \frac{2}{3} C \underset{\sim}{\dot{\alpha}} = \frac{2}{3} C \underset{\sim}{\dot{\varepsilon}}^p - D \underset{\sim}{X} \dot{p} \qquad \dot{R} = b(Q - R)\dot{p}$$

Evaluation of the dissipation



- The intrinsic dissipation is the difference between the plastic power and the power temporarily stored by the hardening mechanisms

$$
\begin{aligned}
\mathcal{D} &= \underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}}^p - \rho \dot{\psi} \\
&= \underset{\sim}{\sigma} : \dot{\underset{\sim}{\varepsilon}}^p - \underset{\sim}{X} : \dot{\underset{\sim}{\alpha}} - R\dot{p} \\
&= \underset{\sim}{\sigma} : \frac{\partial \Omega}{\partial \underset{\sim}{\sigma}} + \underset{\sim}{X} : \frac{\partial \Omega_h}{\partial \underset{\sim}{X}} + R \frac{\partial \Omega_h}{\partial R}
\end{aligned}
$$

$\mathcal{D}$ is automatically positive
iff $\Omega \equiv \Omega_h$ and Ω is convex

Facts concerning ratchetting

- Onedimensional ratchetting
  - Depends on hardening intensity
  - Stopped by linear hardening (either closed or open loop depending on stress range)
  - Too large with non linear hardening
  - Can be adapted by using several kinematic variables, and/or thresholds in the kinematic variables
- Twodimensional ratchetting
  - Depends on hardening intensity *and* direction
  - Too small with linear hardening
  - Too large with classical non linear hardening
  - Can be adapted by a combination of classical non linear hardening and radial fading memory

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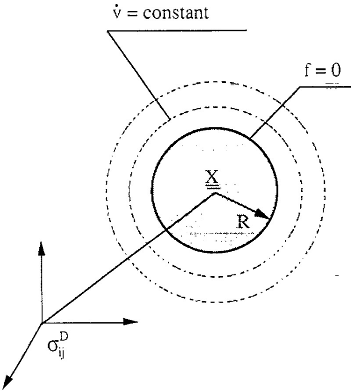
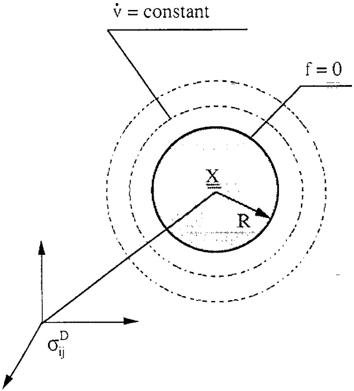
Multimechanism *versus* unified models (1)

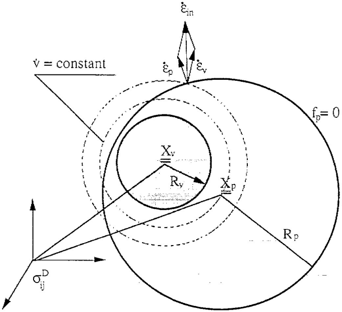


Unified model

Multimechanism *versus* unified models (1)



Unified model



Multimechanism

Example of a multimechanism model with von Mises local criteria

Anatomy of a multimechanism model

- Several sets of potentials, then several components of the plastic strain rate
- Several sets of $(\underset{\sim}{\sigma}^I, \underset{\sim}{X}^I, R^I, \underset{\sim}{\alpha}^I, r^I)$
- Stress $\underset{\sim}{\sigma}^I$ for mechanism I obtained through a concentration tensor $\underset{\approx}{B}^I = \dfrac{\partial \underset{\sim}{\sigma}^I}{\partial \underset{\sim}{\sigma}}$. *For the initial version of the models, $\underset{\approx}{B}^I = \underset{\approx}{I}$ has been chosen*
  - Each $\underset{\sim}{\sigma}^I$ is involved in different yield functions f^I $(\Omega(f^I(\underset{\sim}{\sigma}^I)))$ (2M2C model and crystal plasticity)

$$\dot{\underset{\sim}{\varepsilon}}^p = \sum_I \frac{\partial \Omega^I}{\partial \underset{\sim}{\sigma}} = \sum_I \frac{\partial \Omega^I}{\partial f^I} \frac{\partial f^I}{\partial \underset{\sim}{\sigma}} = \sum_I \frac{\partial \Omega^I}{\partial f^I} \underset{\sim}{n}^I : \underset{\approx}{B}^I \qquad \text{several multiplyiers}$$

  - Each $\underset{\sim}{\sigma}^I$ is involved in a global criterion f $(\Omega(f(\underset{\sim}{\sigma}^I)))$ (2M1C model)

$$\dot{\underset{\sim}{\varepsilon}}^p = \frac{\partial \Omega}{\partial \underset{\sim}{\sigma}} = \frac{\partial \Omega}{\partial f} \frac{\partial f}{\partial \underset{\sim}{\sigma}} = \frac{\partial \Omega}{\partial f} \sum_I \underset{\sim}{n}^I : \underset{\approx}{B}^I \qquad \text{one multiplyier}$$

Each mechanism is either plastic or viscoplastic

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2M2C model : state variables and flow

- 2 mechanisms ($\underset{\sim}{\alpha}^1$, $\underset{\sim}{\alpha}^2$, r^1, r^2), two criteria (yield functions)
- Free energy, depends on $\underset{\sim}{\alpha}^1$, $\underset{\sim}{\alpha}^2$, r^1, r^2 :

$$\rho\psi = \frac{1}{3}\sum_I\sum_J C_{IJ}\underset{\sim}{\alpha}^I : \underset{\sim}{\alpha}^J + \frac{1}{2}\sum_I b_I Q_I \left(r^I\right)^2$$

$$\underset{\sim}{X}^I = \frac{2}{3}\sum_J C_{IJ}\underset{\sim}{\alpha}^J \qquad R^I = b_I Q_I r^I$$

- Potential, sum of two terms :

$$f^I = J(\underset{\sim}{\sigma} - \underset{\sim}{X}^I) - R^I - R_0^I \qquad \Omega = \Omega^1(f^1) + \Omega^2(f^2)$$

$$\dot{\underset{\sim}{\varepsilon}}^p = \frac{\partial\Omega^1}{\partial f^1}\underset{\sim}{n}^1 + \frac{\partial\Omega^2}{\partial f^2}\underset{\sim}{n}^2 \text{ with } \underset{\sim}{n}^I = \frac{\partial f^I}{\partial\underset{\sim}{\sigma}}$$

Each flow can be viscoplastic or plastic

2M1C model : state variables and flow

- 2 mechanisms ($\underset{\sim}{\alpha}^1$, $\underset{\sim}{\alpha}^2$, r), one criterion (yield function)
- Free energy, depends on $\underset{\sim}{\alpha}^1$, $\underset{\sim}{\alpha}^2$, r :

$$\rho\psi = \frac{1}{3}\sum_I \sum_J C_{IJ} \underset{\sim}{\alpha}^I : \underset{\sim}{\alpha}^J + \frac{1}{2} bQr^2$$

$$\underset{\sim}{X}^I = \frac{2}{3}\sum_J C_{IJ} \underset{\sim}{\alpha}^J \qquad R = bQr$$

- Potential, depends on one yield only :

$$f = \left(J(\underset{\sim}{\sigma} - \underset{\sim}{X}^1)^2 + J(\underset{\sim}{\sigma} - \underset{\sim}{X}^2)^2\right)^{1/2} - R - R_0 \qquad \Omega \equiv \Omega(f)$$

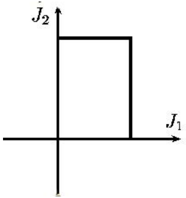
$$\underset{\sim}{\dot{\varepsilon}}^p = \frac{\partial \Omega}{\partial f} \underset{\sim}{n} \qquad \text{with} \qquad \underset{\sim}{n} = \frac{J_1 \underset{\sim}{n}^1 + J_2 \underset{\sim}{n}^2}{(J_1^2 + J_2^2)^{1/2}}$$

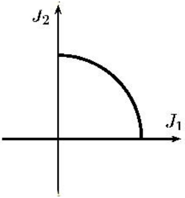
$$\text{and } J_I = J(\underset{\sim}{\sigma} - \underset{\sim}{X}^I) \qquad \underset{\sim}{n}^I = \frac{3}{2}\frac{\underset{\sim}{s} - \underset{\sim}{X}^I}{J^I}$$

Only one (visco)plastic flow

Comparison between 2M2C and 2M1C model features

- Application of the normality rule





- Coupling between the hardening variables

$$\begin{pmatrix} \underset{\sim}{X}_1 \\ \underset{\sim}{X}_2 \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{pmatrix} \begin{pmatrix} \underset{\sim}{\alpha}_1 \\ \underset{\sim}{\alpha}_2 \end{pmatrix}$$

- *Ratchetting iff the determinant* $C_{11}C_{22} - C_{12}^2 = 0$

2M2C model : hardening

- Hardening rules :

$$\dot{\underset{\sim}{\alpha}}^I = \left(\underset{\sim}{n}^I - \frac{3D_I}{2C_{II}} \underset{\sim}{X}^I \right) \frac{\partial \Omega^I}{\partial f^I} \qquad \dot{r} = \left(1 - \frac{R^I}{Q_I} \right) \frac{\partial \Omega^I}{\partial f^I}$$

- Denote $\dot{p}^I = \frac{\partial \Omega^I}{\partial f^I}$ for plastic, and $\dot{v}^I = \frac{\partial \Omega^I}{\partial f^I}$ for viscoplastic flow
- Three possible models :

2M2C-VV , two viscoplastic strains $\dot{\underset{\sim}{\varepsilon}}^p = \dot{v}^1 \underset{\sim}{n}^1 + \dot{v}^2 \underset{\sim}{n}^2$
2M2C-PP , two plastic strains $\dot{\underset{\sim}{\varepsilon}}^p = \dot{p}^1 \underset{\sim}{n}^1 + \dot{p}^2 \underset{\sim}{n}^2$
2M2C-VP , one plastic, one viscoplastic $\dot{\underset{\sim}{\varepsilon}}^p = \dot{v}^1 \underset{\sim}{n}^1 + \dot{p}^2 \underset{\sim}{n}^2$

2M2C-PP model

For two mechanisms :

$$\dot{\lambda}^1 = \left\langle \frac{M_{22}\,\underset{\sim}{n}^1 : \dot{\underset{\sim}{\sigma}} - M_{12}\,\underset{\sim}{n}^2 : \dot{\underset{\sim}{\sigma}}}{M_{11}M_{22} - M_{12}M_{21}} \right\rangle$$

$$\dot{\lambda}^2 = \left\langle \frac{M_{11}\,\underset{\sim}{n}^2 : \dot{\underset{\sim}{\sigma}} - M_{21}\,\underset{\sim}{n}^1 : \dot{\underset{\sim}{\sigma}}}{M_{11}M_{22} - M_{12}M_{21}} \right\rangle$$

with

$$M_{II} = C_{II} - D_I \underset{\sim}{X}^I : \underset{\sim}{n}^I + b_I(Q_I - R^I)$$

$$M_{IJ} = \frac{2}{3}\, C_{IJ}\,\underset{\sim}{n}^I : \underset{\sim}{n}^J - D_J \frac{C_{IJ}}{C_{JJ}}\,\underset{\sim}{n}^I : \underset{\sim}{X}^J$$

2M2C-VP model (1)

Viscoplastic part $(1 \equiv v)$

$$f^v = J(\underset{\sim}{\sigma} - \underset{\sim}{X}^v) - R^v - R_{ov}$$
$$\underset{\sim}{X}^v = (2/3) C_v \underset{\sim}{\alpha}^v + C_{vp} \underset{\sim}{\alpha}^p$$
$$R^v = b_v Q_v r^v$$
$$\dot{\underset{\sim}{\alpha}}^v = \dot{\underset{\sim}{\varepsilon}}^v - \left(\frac{3D_v}{2C_v}\right) \underset{\sim}{X}^v \dot{v}$$
$$\dot{r}^v = \left(1 - \frac{R^v}{Q_v}\right) \dot{v}$$
$$\dot{v} = \left\langle \frac{f^v}{K} \right\rangle^n$$

Plastic part $(2 \equiv p)$

$$f^p = J(\underset{\sim}{\sigma} - \underset{\sim}{X}^p) - R^p - R_{op}$$
$$\underset{\sim}{X}^p = (2/3) C_p \underset{\sim}{\alpha}^p + C_{vp} \underset{\sim}{\alpha}^v$$
$$R^p = b_p Q_p r^p$$
$$\dot{\underset{\sim}{\alpha}}^p = \dot{\underset{\sim}{\varepsilon}}^p - \left(\frac{3D_p}{2C_p}\right) \underset{\sim}{X}^p \dot{\lambda}$$
$$\dot{r}^p = \left(1 - \frac{R^p}{Q_p}\right) \dot{p}$$
$$\dot{\lambda} = \frac{< \dot{\underset{\sim}{\sigma}} - C_{vp} \dot{\underset{\sim}{\alpha}}^v >: \underset{\sim}{n}^p}{H_p}$$

with $H_p = C_p - D_p \underset{\sim}{X}^p : \underset{\sim}{n}^p + b_p (Q_p - R^p)$

2M2C-VP model (2)

Remark : consistency condition for 2M2C-VP model

$$\dot{f}^p = \underset{\sim}{n}^p : \underset{\sim}{\dot{\sigma}} - \underset{\sim}{n}^p \underset{\sim}{\dot{X}}^p - \dot{R}^p$$

Coupling through :

$$\underset{\sim}{\dot{X}}^p = C_{pp} \underset{\sim}{\dot{\alpha}}^{pp} + C_{vp} \underset{\sim}{\dot{\alpha}}^{vp}$$

$$\underset{\sim}{\dot{X}}^p = C_{pp} \left(\underset{\sim}{n}^p - \frac{3D_p}{2C_{pp}} \underset{\sim}{X}^p \right) \dot{p} + C_{vp} \left(\underset{\sim}{n}^v - \frac{3D_v}{2C_{vv}} \underset{\sim}{X}^v \right) \dot{v}$$

$$\dot{R}^p = b_p Q_p \left(1 - \frac{R^p}{Q_p} \right) \dot{p}$$

The plastic increment $\dot{p}$ is now time dependent...

2M2C-VP model(3)

- Limitation of the viscous stress for very high strain rate by the time independent model
- The yield limit for creep can be much lower than yield limit for inviscid plasticity
- Full/limited coupling between plasticity and creep
- Special coefficient sets provide *inverse strain rate* effect (lower stress for higher strain rate)

2M1C-P : plastic multiplyier

$$\dot{\lambda} = \left\langle \frac{(J_1 \underset{\sim}{n}^1 + J_2 \underset{\sim}{n}^2) : \dot{\underset{\sim}{\sigma}}}{h_R + h_{X1} + h_{X2}} \right\rangle$$

with :

$$h_R = b(Q - R)R$$

$$h_{X1} = \frac{2}{3}\left(\underset{\sim}{n}^1 - \frac{3D_1}{2C_{11}}\underset{\sim}{X}^1\right) : \left(C_{11}J_1\underset{\sim}{n}^1 + C_{12}J_2\underset{\sim}{n}^2\right)$$

$$h_{X2} = \frac{2}{3}\left(\underset{\sim}{n}^2 - \frac{3D_2}{2C_{22}}\underset{\sim}{X}^2\right) : \left(C_{12}J_1\underset{\sim}{n}^1 + C_{22}J_2\underset{\sim}{n}^2\right)$$

No ratchetting, except if $C_{11}C_{22} - C_{12}^2 = 0$

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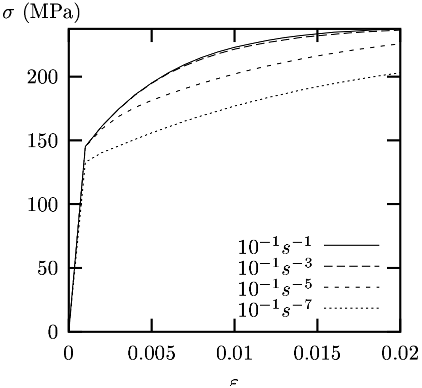
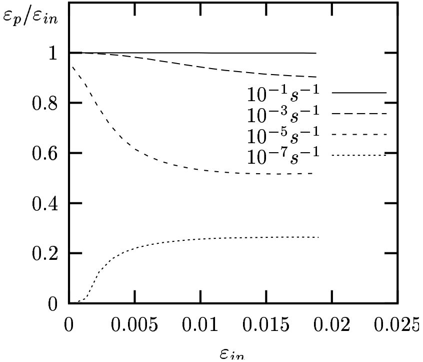
2M2C-VP model : Balance between plastic and viscoplastic strain



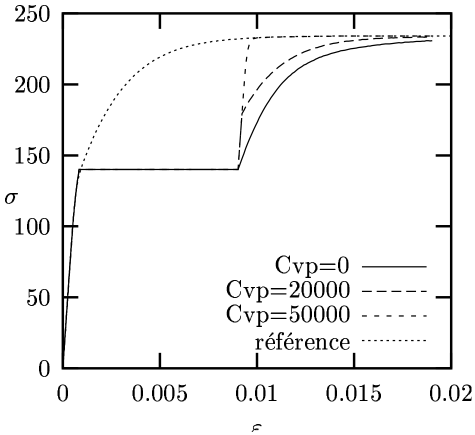
Tensile curves at various strain rates



Amount of plasticity in inelastic strain

$$K = 500; n = 7; R_0^v = 80; C_v = 10000; D = 100; R_0^p = 140; C_p = 20000; D = 200$$

2M2C-VP model : Influence of the coupling term

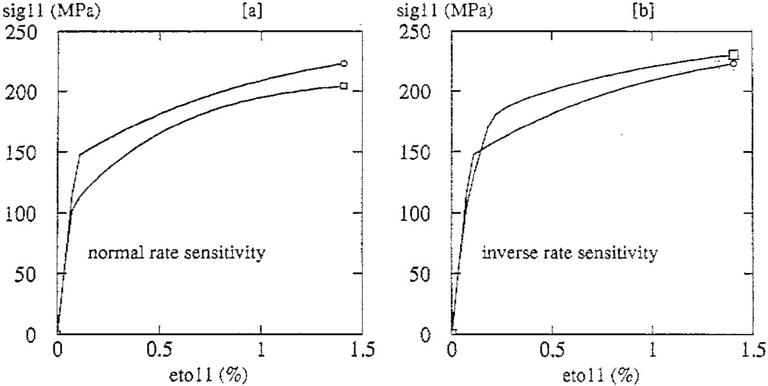


(units : MPa, s)
viscosity :
K = 500 ; n = 7
isotropic viscous :
R_0 = 0
kinematic viscous :
C = 50000 ; D = 500
isotropic plastic :
R_0 = 140
kinematic plastic :
C = 50000 ; D = 500

Creep 555 h at 140 MPa, then tension at $\dot{\varepsilon} = 10^{-4}s^{-1}$
reference in tension at $\dot{\varepsilon} = 10^{-4}s^{-1}$

2M2C-VP model : Inverse strain rate effect

○ $\dot{\varepsilon} = 10^{-3} s^{-1}$ □ $\dot{\varepsilon} = 10^{-6} s^{-1}$

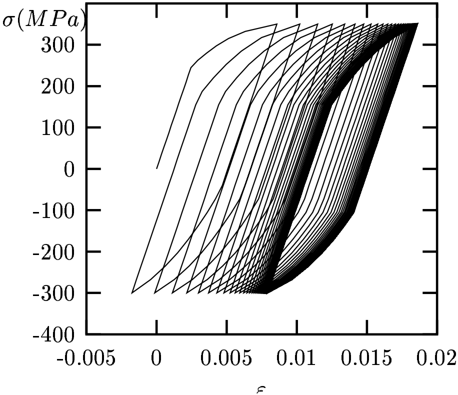


Normal rate sensitivity if $C_p < C_v < C_{vp}$
$C_p = 10000 < C_v = 20000 < C_{vp} = 40000$
$D_p = 100, D_v = 200$

Inverse rate sensitivity if $C_p < C_{vp} \ll C_v$
$C_p = 10000 < C_{vp} = 40000 < C_v = 100000$
$D_p = 400, D_v = 100$

Other coefficients : $K = 700, n = 7.2; R_0^v = 0; R_0^p = 140$

2M1C-P : Plastic shakedown



coefficients (MPa,s)
viscosité :
n=2, K=5
cinématique I :
C_{11} = 2000 ; D_1 = 0
cinématique II :
C_{22}= 200000 ; D_2 = 0
isotrope :
R_0 = 350
couplage :
C_{12} = -20000

Steady state presenting an open loop under non symmetrical loading

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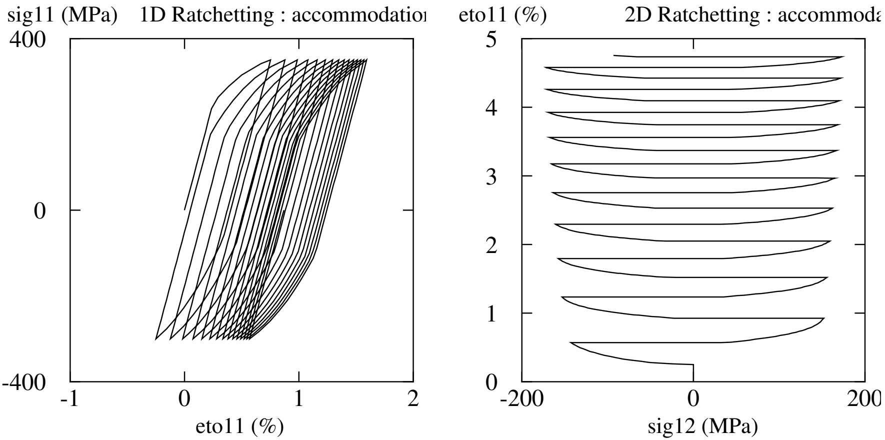
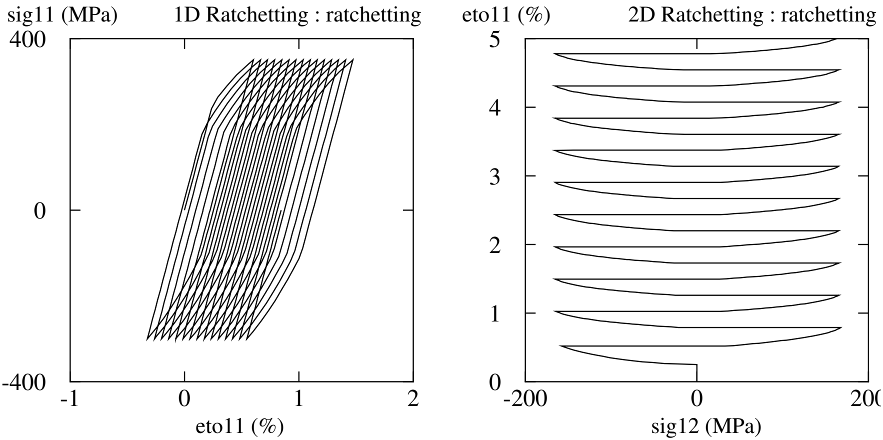
Ratchetting behavior, Regular Matrix



No ratchetting since the determinant $C_{11}C_{22} - C_{12}^2 \neq 0$

Ratchetting behavior, Singular Matrix



Ratchetting since the determinant $C_{11}C_{22} - C_{12}^2 = 0$

Possible improvements

- The ratchetting behavior in the multimechanism models is the result of :
  - The character of the hardening matrix (Regular or Singular)
  - The evolution rules of the kinematic hardening variables (Linear or Non Linear)
- The source of improvements are :
  - The localization rules of the micro-mechanical (β–rule)
  - The evolution rules etablished for the unified models

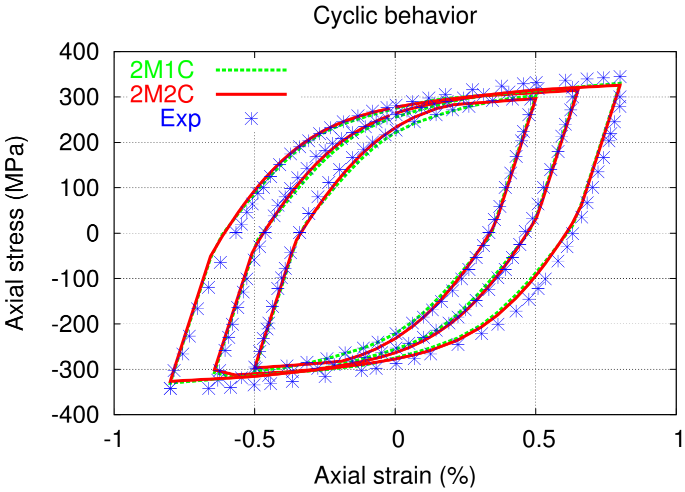
An identification of the model

Experimental data base of a 316 stainless steel ([Portier et al., 2000])
tests at room temperature (25°C)

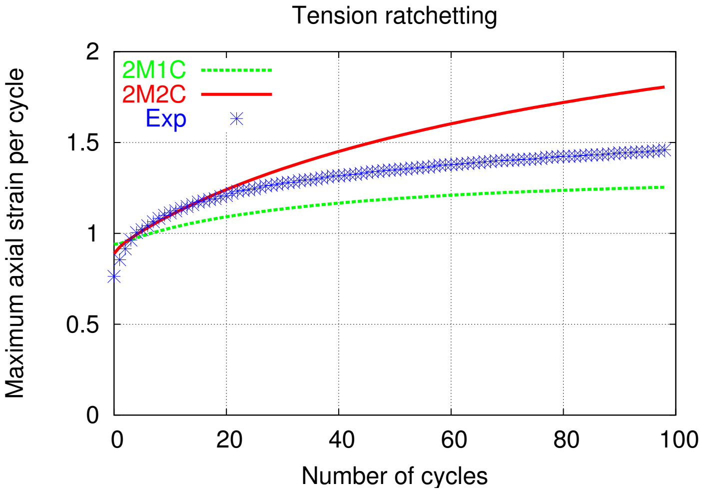
- Monotonic tensile tests,
- Cyclic uni-axial tension-compression for three strain ranges,
- Tension-torsion ratchetting tests with two values of tensile stress and with different shear strain amplitude,
- Tension-torsion out-of-phase test in the steady-state stress response.

2M1C_β and 2M2C_β have been identified

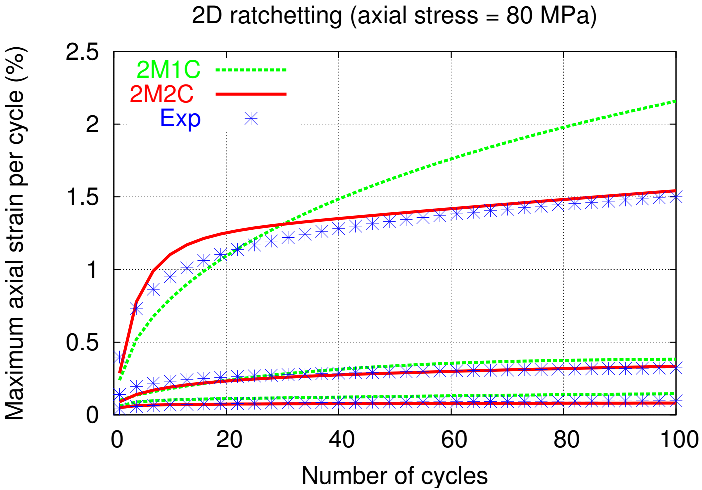
Cyclic tests



Onedimensional ratchetting

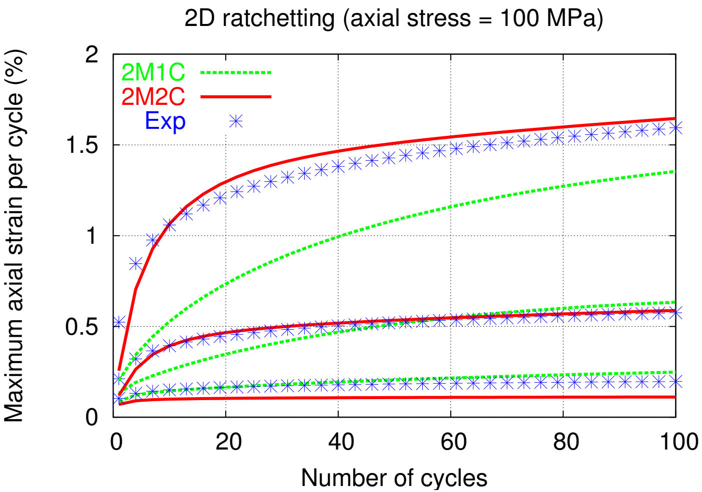


Twodimensional ratchetting (1)



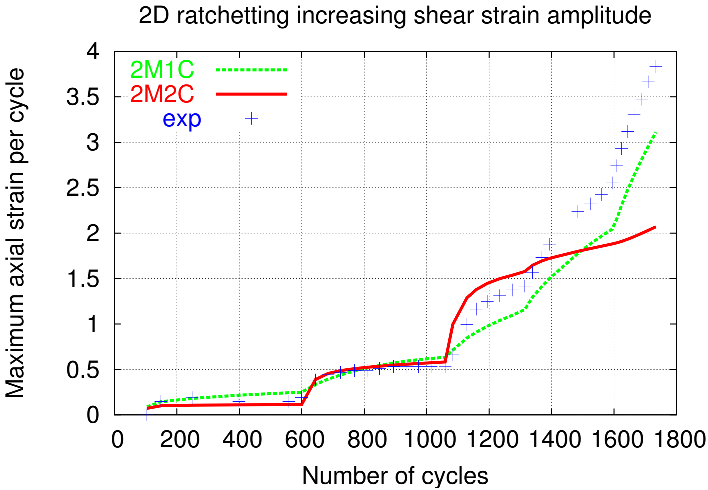
Axial stress 80 MPa

Twodimensional ratchetting (2)

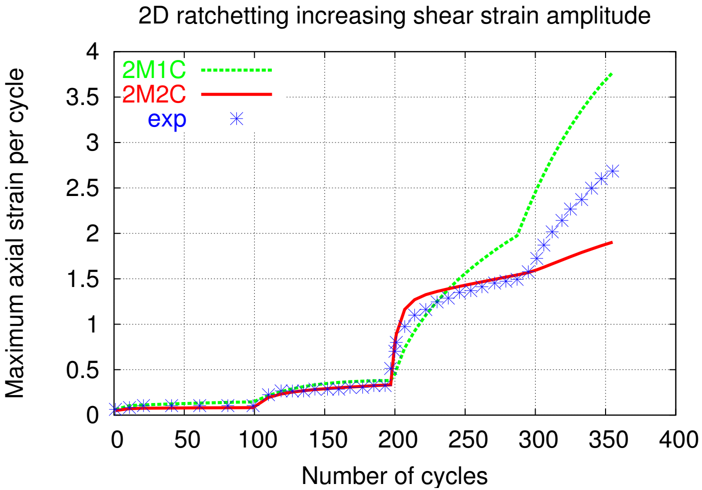


Axial stress 100 MPa

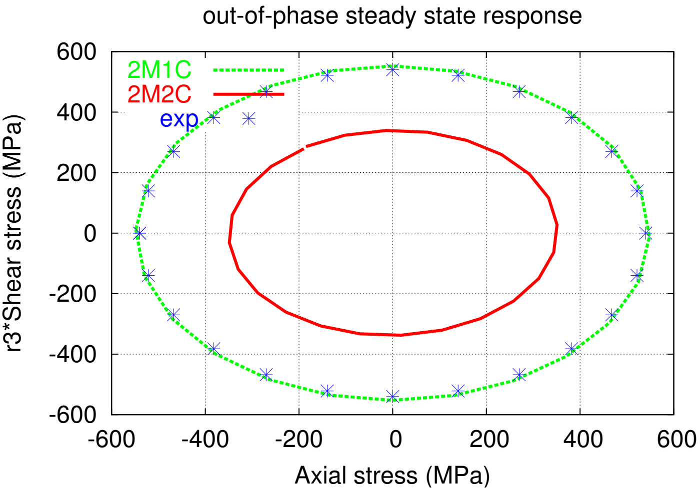
2D ratchetting, increasing strain amplitude (1)



2D ratchetting, increasing strain amplitude (2)



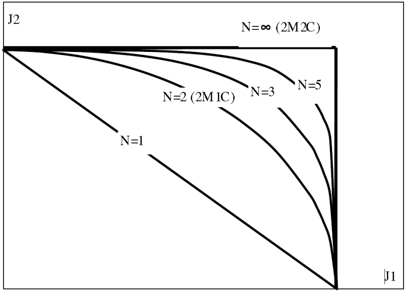
Out-of-phase test



Concluding remarks

- Other versions are possible in the *nMmC* model class
- Fully equipped with singular and regular/singular matrices
- Choices of fading memory terms, strain memory effect, etc.
- High versatility *wrt* ratchetting behavior, additional hardening
- Several groups are involved in new developments
  - K. Sai (ENIS Sfax, Tunisia) [Cailletaud and Saï, 1995, Sai and Cailletaud, 2006, Saï and Cailletaud, 2007, Cailletaud and Saï, 2008, Saï et al., 2012, Saï et al., 2014]
  - L. Taleb (INSA Rouen, France) [Taleb et al., 2006, Taleb and Cailletaud, 2010, Taleb and Cailletaud, 2011, Saï et al., 2014]
  - M. Wolff (Univ. Bremen, Germany) [Wolff and Taleb, 2008]

Alternative form of 2M1C's criterion



- Instead of

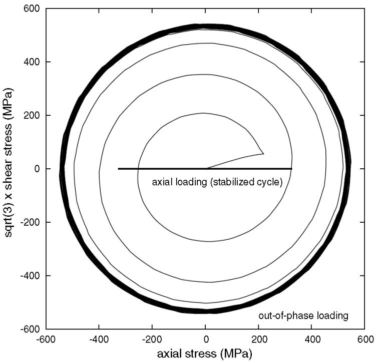
$$f = \left(J(\underset{\sim}{\sigma} - \underset{\sim}{X}^1)^2 + J(\underset{\sim}{\sigma} - \underset{\sim}{X}^2)^2\right)^{1/2} - R - R_0 = \left(J_1^2 + J_2^2\right)^{1/2} - R - R_0$$

- Write

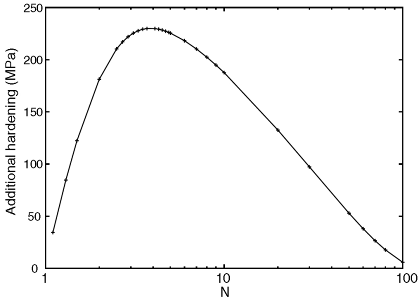
$$f = \left(J(\underset{\sim}{\sigma} - \underset{\sim}{X}^1)^N + J(\underset{\sim}{\sigma} - \underset{\sim}{X}^2)^N\right)^{1/N} - R - R_0 = \left(J_1^N + J_2^N\right)^{1/N} - R - R_0$$

- Continuous transition from 2M1C ($N = 2$) to 2M1C ($N \to \infty$)

Influence of N on additional hardening



Stress response for 90°phase lag in strain space



Additional hardening versus N

Maximum for $N = 3$, optimum wrt experiment, $N = 2$ (or $N = 4$)

Alternative form of the fading memory term

- Instead of using $\underset{\sim}{X}^I$,

$$\dot{\underset{\sim}{\alpha}}^I = \left(\underset{\sim}{n}^I - \frac{3D_I}{2C_{II}}\underset{\sim}{X}^I\right)\dot{p}^I$$

1/ Use $\underset{\sim}{\alpha}^I$

$$\dot{\underset{\sim}{\alpha}}^I = \left(\underset{\sim}{n}^I - D_I\underset{\sim}{\alpha}^I\right)\dot{p}^I$$

(the thermodynamical aspect is then more difficult to achieve)

2/ Use $(1-\eta)\alpha^I + \eta(\alpha^I : \underset{\sim}{n}^I)\underset{\sim}{n}^I$

$$\dot{\underset{\sim}{\alpha}}^I = \left(\underset{\sim}{n}^I - D_I\left[(1-\eta)\alpha^I + \eta(\alpha^I : \underset{\sim}{n}^I)\right]\right)\dot{p}^I$$

(one more parameter, η, to adjust the direction of the fading memory term)

Warning : thermodynamics must be respected

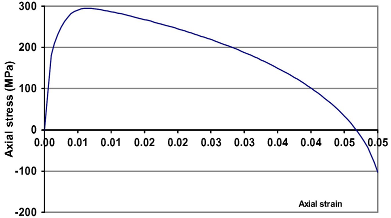
- General condition

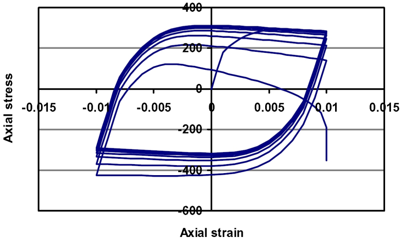
$$C_{11}C_{22} - C_{12}^2 \geqslant 0$$

- In case of a fading memory with η

$$\eta \leqslant 1 - \sqrt{\frac{C_{12}^2 (D_1 + D_2)^2}{4 C_{11} C_{22} D_1 D_2}}$$

- Example with $C_{11}C_{22} - C_{12}^2 < 0$





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