TD2 EX1 : Compute forces on S_u

1- Matlab code lines to compute forces $\underline{R} = R_1 \underline{e}_1 + R_2\, \underline{e}_2$ on S_u

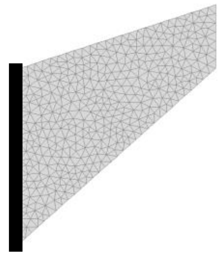
with, by definition $\underline{R} = \int_{S_u} \underline{\underline{\sigma}}.\underline{n}\, dS$ and $R_1 = \left(\int_{S_u} \underline{\underline{\sigma}}.\underline{n}\, dS \right).\underline{e}_1$ $R_2 = \left(\int_{S_u} \underline{\underline{\sigma}}.\underline{n}\, dS \right).\underline{e}_2$

2- Applications: wing & compress

Wing case:

on S_u (x=0)

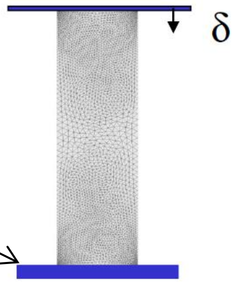
(R_1, R_2) = ?



Compress case:

on S_u (y=0)

(R_1, R_2) = ?



Hints :TD2 EX1 : Compute forces on S_u

1- Matlab code lines to compute forces $\underline{R} = R_1\underline{e}_1 + R_2\,\underline{e}_2$ on S_u

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Indication:

Use PPV (book: eq. 1.19) with a well-chosen test field $\underline{w}_h \in C$

$$\int_\Omega \underline{\underline{\varepsilon}}[\underline{u}_h] : \boldsymbol{\mathcal{A}} : \underline{\underline{\varepsilon}}[\underline{w}_h]\, \mathrm{d}V = \int_\Omega \rho \underline{f}.\underline{w}_h\, \mathrm{d}V + \int_{S_T} \underline{T}^{\mathrm{D}}.\underline{w}_h\, \mathrm{d}S + \int_{S_u} [\underline{\underline{\sigma}}.\underline{n}].\underline{w}_h\, \mathrm{d}S$$

To find nodes on S_u == > use matlab function « find »

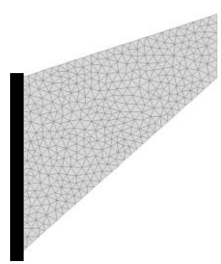
S_nodes = find(coor(:,1)<(1.d-4)); == > S_nodes returns the indices of « coor(:,1) » that satisfy the condition

2- Applications: wing & compress

Wing case:

on S_u (x=0)

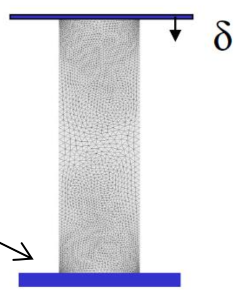
(R_1, R_2) = ?



Compress case:

on S_u (y=0)

(R_1, R_2) = ?

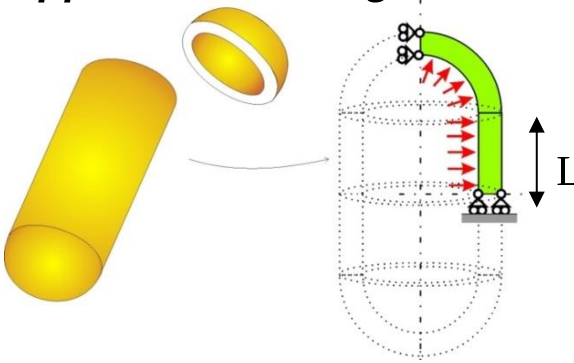


TD2 EX2 Axisymmetric element (T6axi)

Axisymmetric problems :
+ 3D geometry is completely represented by a "2D" mesh
+ Nothing depends on θ

== > axisymmetric formulation

Application: diving bottle



1- Writes PPV (TTV) in function of r and z

$$\underline{u}(r,z) = u_r(r,z)\underline{e}_r + u_z(r,z)\underline{e}_z$$

$$\underline{\underline{\varepsilon}}(\underline{u}) = u_{r,r}\,\underline{e}_r \otimes \underline{e}_r + u_{z,z}\,\underline{e}_z \otimes \underline{e}_z + \frac{u_r}{r}\,\underline{e}_\theta \otimes \underline{e}_\theta + \frac{1}{2}\left(u_{r,z} + u_{z,r}\right)\left(\underline{e}_r \otimes \underline{e}_z + \underline{e}_z \otimes \underline{e}_r\right)$$

2- Definition of {ε} et {σ} == > [A] : {σ}=[A]{ε}
3- Definition of [A] == > utility/read_input.m (line 263)
4- Create in T6axi directory 3 functions:
- stiff_linel_T6axi.m (local stiffness matrix)
- nf_tractions_T6axi.m (local generalized force vector)
- stressG_linel_T6axi.m (Cauchy stress component at Gauss points)

5- Analysis / analytical comparaison

Geometry:
R_I = 10, R_E = 11, L = 20
Loading:
internal pressure p=1
Input File :
input/reservoir.geo,
input/reservoir.inp

Analytical solution:
Sphere under pressure:

$$u_r = A\frac{(1-2\nu)}{E}r + B\frac{1+\upsilon}{E}\frac{1}{r^2}, \quad A = p\frac{R_I^3}{R_E^3 - R_I^3}, \quad B = \frac{p}{2}\frac{R_I^3 R_E^3}{R_E^3 - R_I^3}$$

Cylinder under pressure:

$$u_r = A\frac{(1-2\nu)}{E}r + B\frac{(1+\upsilon)}{E}\frac{1}{r}, \quad A = p\frac{R_I^2}{R_E^2 - R_I^2}, \quad B = p\frac{R_I^2 R_E^2}{R_E^2 - R_I^2}$$

A.N. : $U_{\text{sphère}}$ = 38.2

U_{cylindre} = 94

Send by email: (clement.olivier@safran.fr)
- ***Your names***
- ***4 matlab programs:* stiff_linel_T6axi.m, nf_tractions_T6axi.m, stressG_linel_T6axi.m *1-2 pages (max) of comments and analysis***