

PLASTICITÉ CRISTALLINE ET TRANSITION D'ÉCHELLE : CAS DU MONOCRISTAL

—
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1. Introduction & motivation

2. Physical origins of crystal slip

Experimental observations

Taylor's paradox

Dislocations

3. Continuum crystal plasticity

The schmid law

Geometry of slip and Schmid factors in FCC crystals

Constitutive rules for single crystal plasticity

Example of monocrystal behaviour

Example of FE computation



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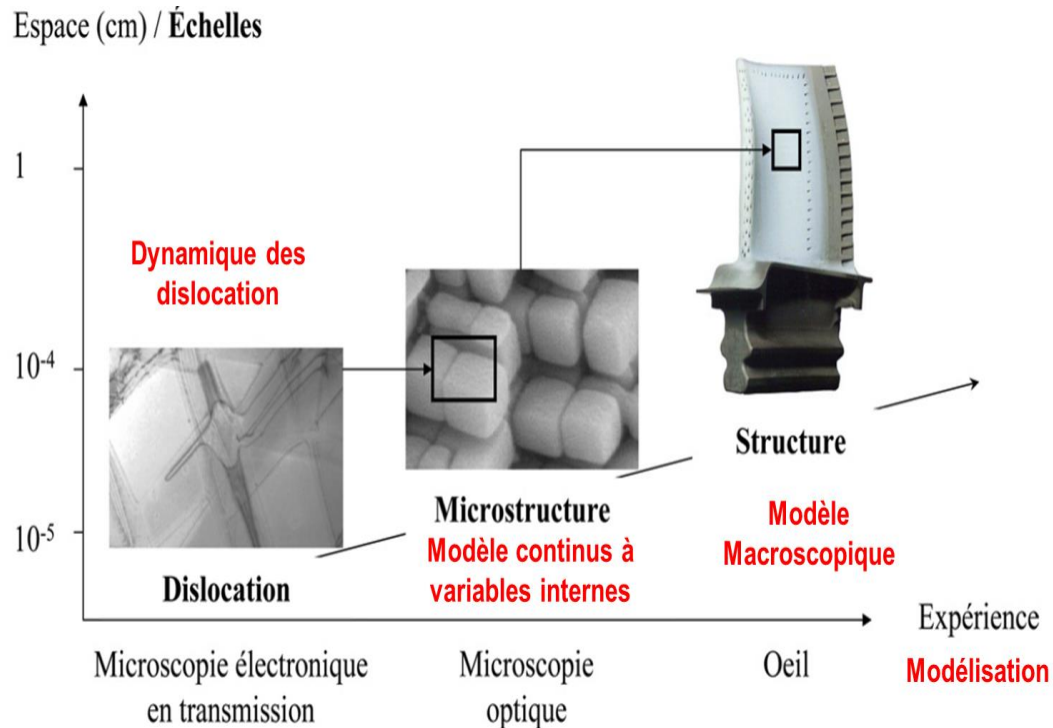
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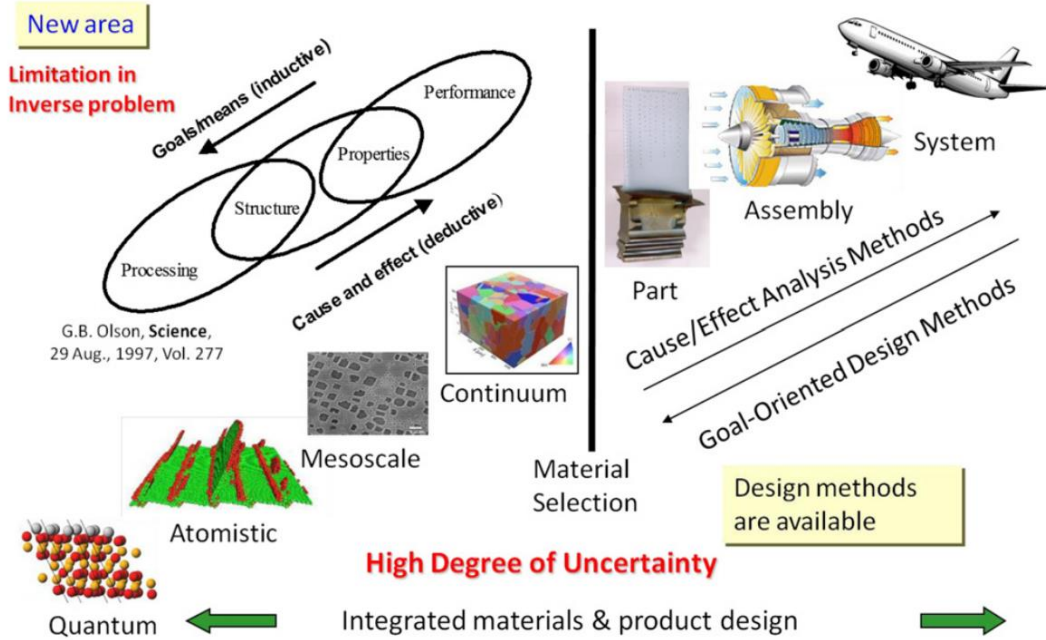
Example of FE computation

Deformation of crystalline materials : a multiscale problem



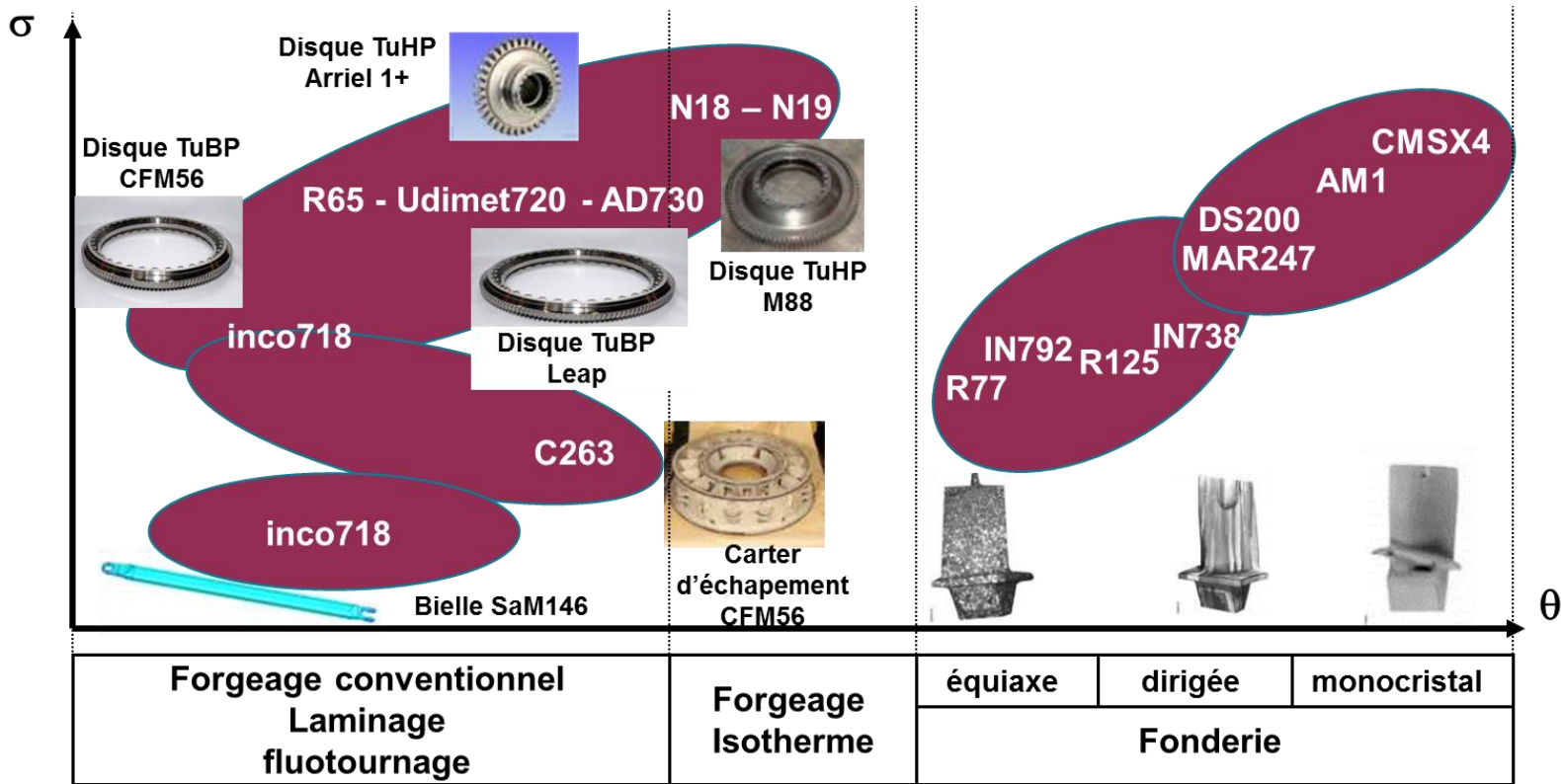
Plastic deformation results in the motion of lots of defects in the crystal lattice (dislocations)
→ continuum scale ($> \mu\text{m}$)

Integrated Computational Materials Engineering

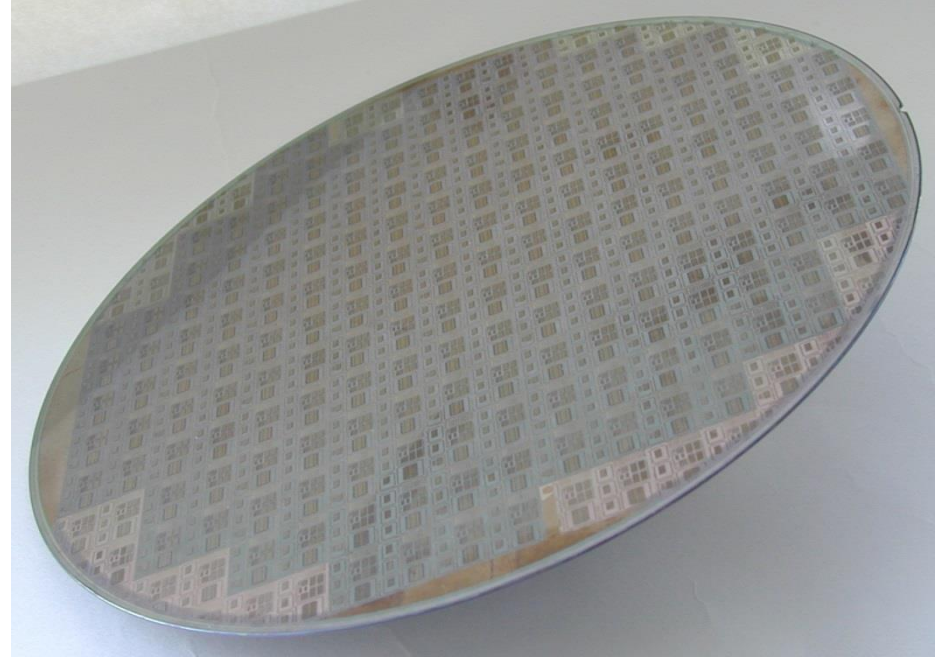
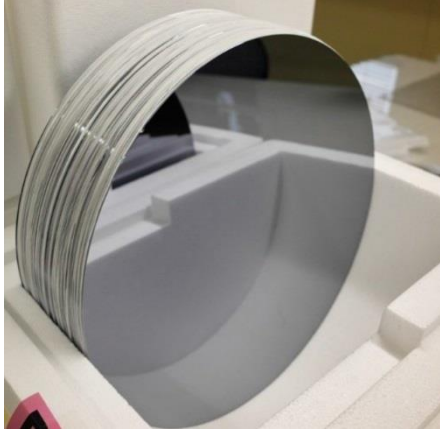


Local values (as opposed to mean values) are crucial when studying deformation and fracture → model for crystal plasticity needed.

Such applications in aeronautic (Safran sources)



Such applications in electronics (Single crystal)





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A brief history



V. Volterra

1860 - 1940



G.I. Taylor

1886 - 1975



M. Polanyi

1891 - 1976



E. Orowan

1891 - 1976

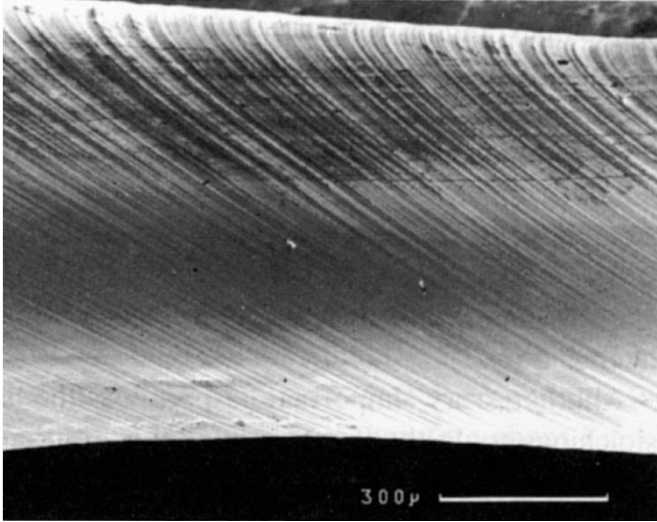


P.B. Hirsch

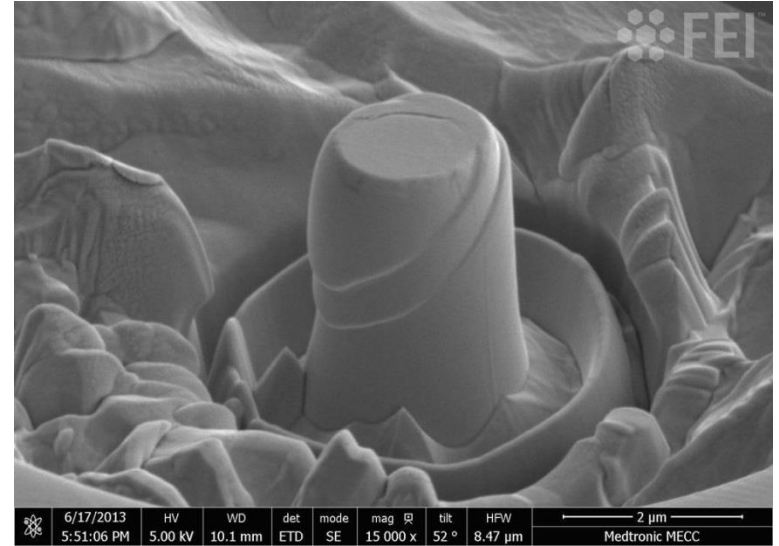
1920 -

The early of the history of dislocation theory. 1907: Definition of Volterra's dislocation (or isolated defect) by the mathematician V. Volterra; 1934: Discovery of the concept of crystal dislocation introduced by Orowan, Polany and Taylor; 1956: First published observations of dislocations by transmission electron microscopy.

60 years of plastic slip in monocrystals



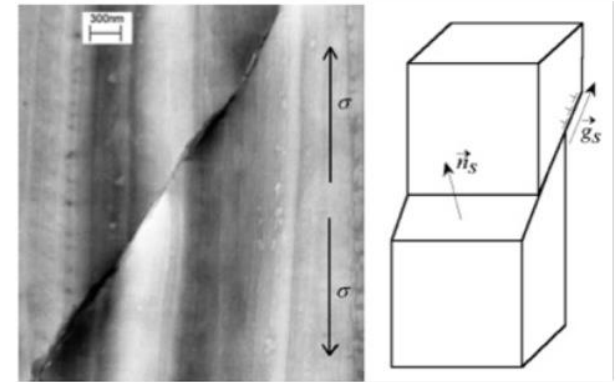
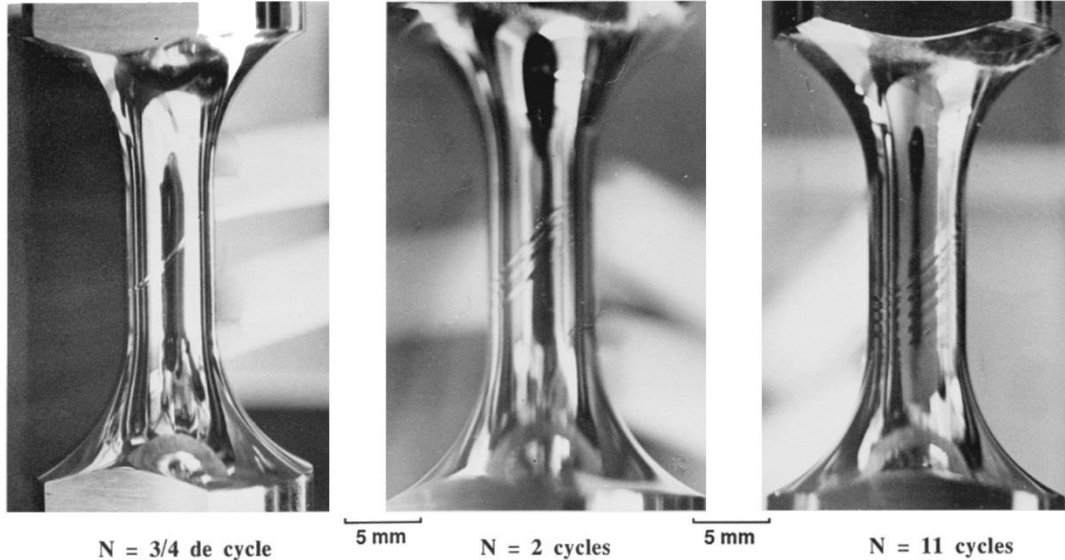
Cadmium monocrystal deformed under tension
[Schmid and Boas, 1950]



Fib fabricated micropillar deformed under
compression [Shan et al., 2007]

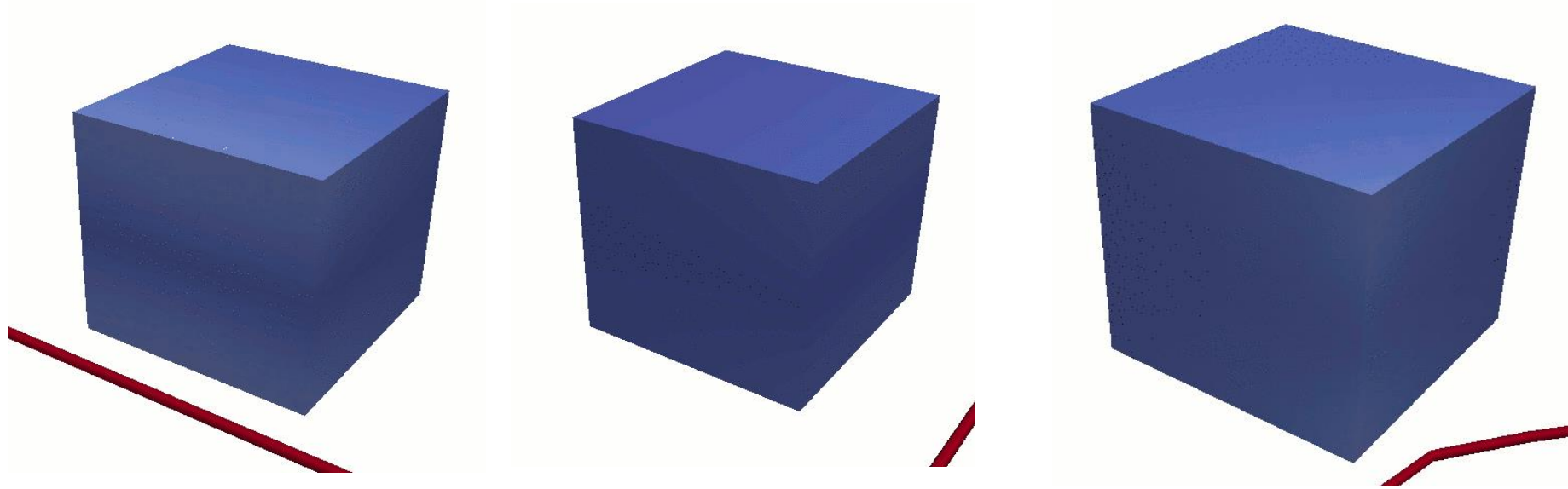
Conclusion : Crystals deform by shear. Shear appear to be localized on the highest atomic density planes, the so-called *slip planes*.

Evidences of crystallographic slip in Ni-based superalloy



With each passage of a dislocation through a single crystal corresponds the appearance of a "walk" on the surface and a shift of the two parts. The set of these shifts generates the plastic deformation observable on the macroscopic scale

The concept of dislocation (1/2)



pure edge

pure screw

mixed dislocation

The concept of dislocation (2/2)

A dislocation can be represented by :

- ◆ a line vector L
- ◆ a Burgers vector b

Dislocations may pre-exist and are created during plastic deformation.

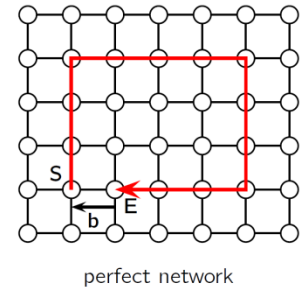
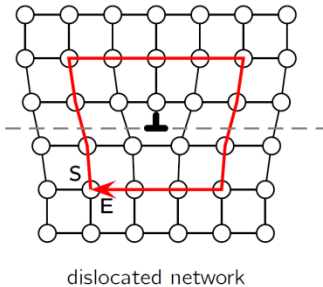
- ◆ The sum of the Burgers vector of created dislocations is equal to zero

A dislocation moves

- ◆ The motion is defined by a glide plane (L , b) and a glide direction g
- ◆ Screw dislocation don't have a single glide plane
- ◆ Edge dislocation do have a single glide plane
- ◆ Plastic deformation is associated to the flux of disloc-

The crystal is distorted around dislocations

- ◆ Dislocations are stress concentrators
- ◆ There is an elastic energy associated to dislocations

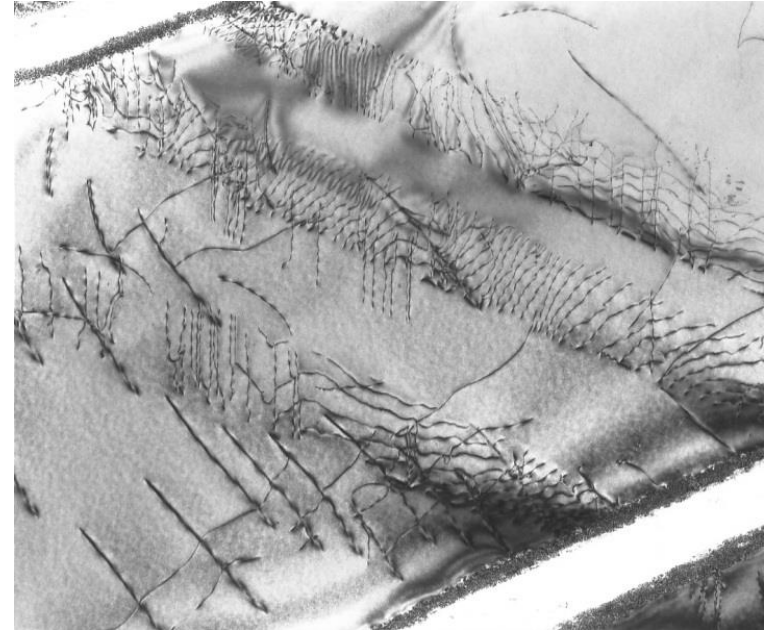


Can we actually see dislocations ?



Microscope FEI TECNAI F20-ST
Centre des Matériaux – MINES
ParisTech

Bright field image
Dislocation microstructure of
deformed Ti-6242 alloy

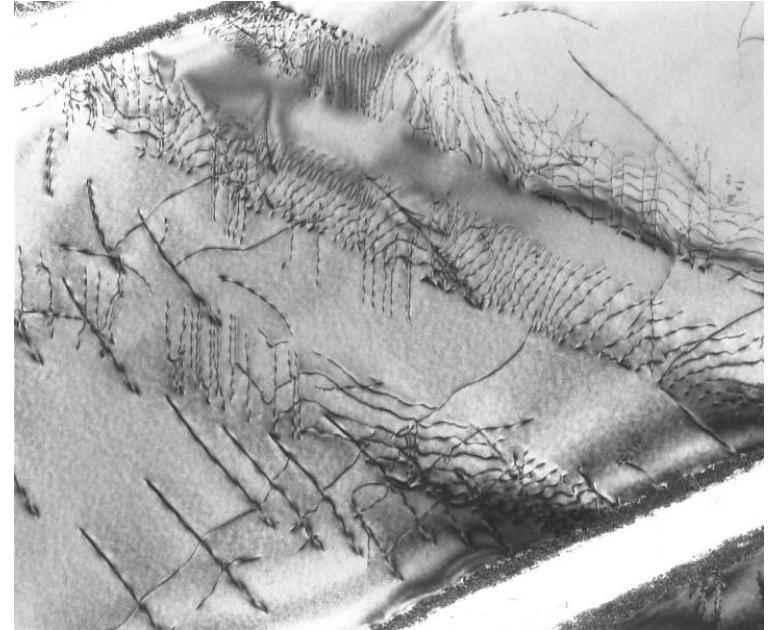


Can we actually see dislocations ?

In 1cm³ of metal alloys (order of magnitude):

- 10m of dislocation in a single crystal alloy
- 10.000km in a polycrystal weakly hardened
- 10.000.000km in a highly hardened polycrystal

Bright field image
Dislocation microstructure of
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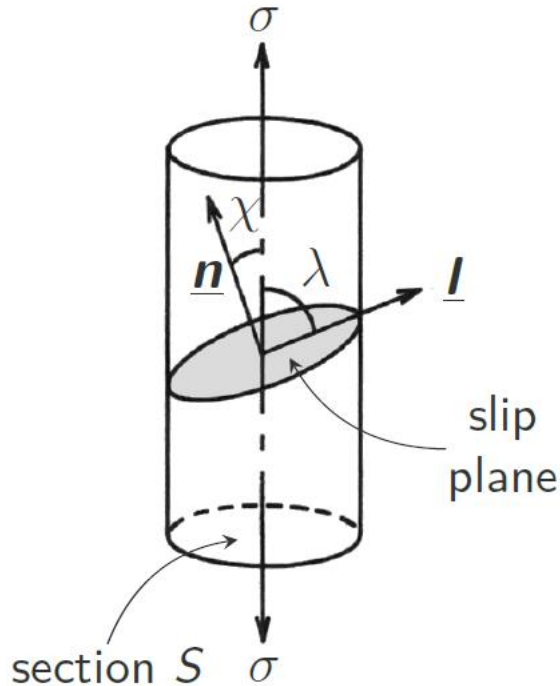
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Tensile test applied to SX - Resolved shear stress



Given a slip direction \underline{l} contained in a slip plane with a normal vector \underline{n} , the shear τ in the slip direction can be related to the applied tensile stress σ :

$$\tau = \frac{F \cos(\lambda)}{S / \cos(\chi)} = \sigma \cos(\lambda) \cos(\chi)$$

$m = \cos(\lambda) \cos(\chi)$ [0, 0.5] is called the **Schmid factor**.

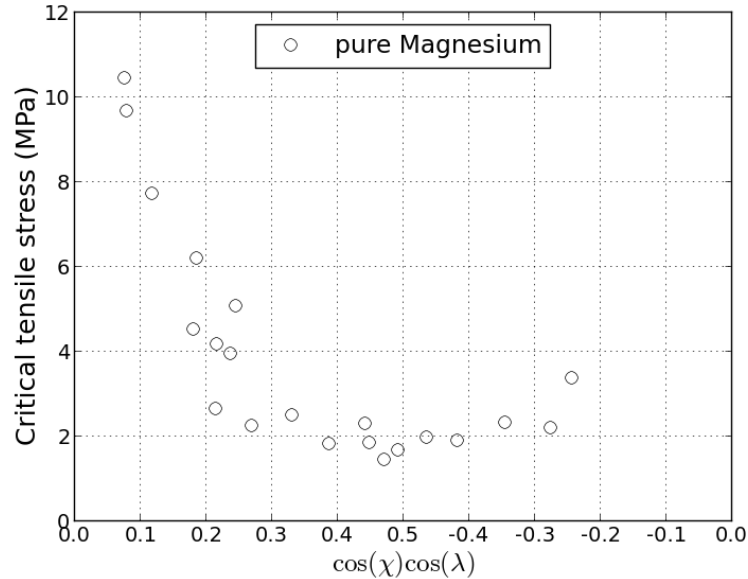
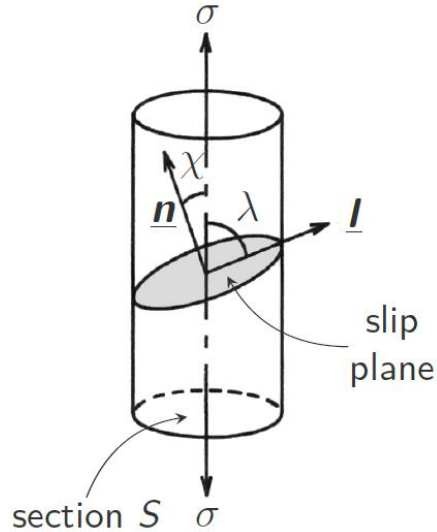
Generalization, in tensorial form we have :

$$\tau = l_i \sigma_{ij} n_j$$

The critical resolved shear stress

Experiments of Schmid and Boas

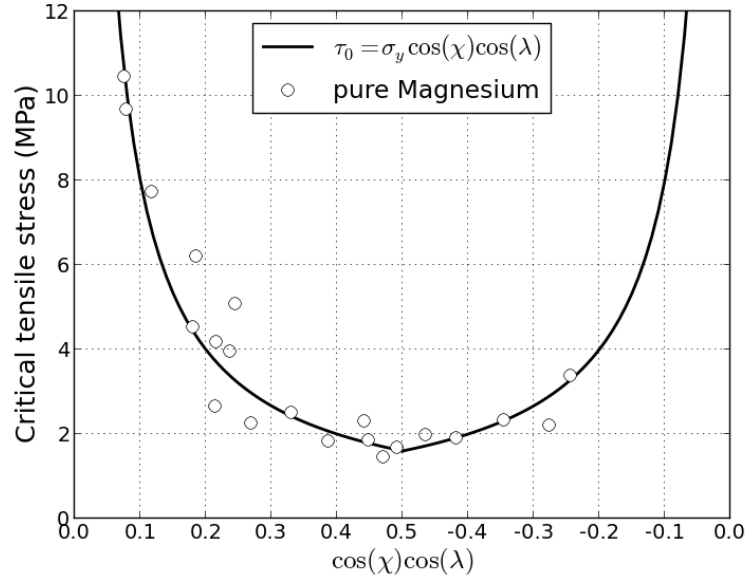
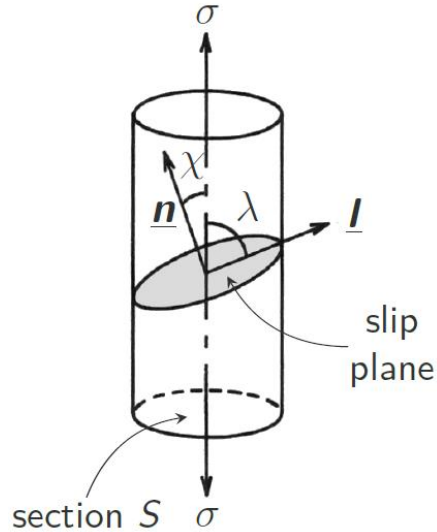
- ◆ No influence of the normal stress operating on the glide plane.
- ◆ A critical value of the resolved shear stress is required for the initiation of glide.



The critical resolved shear stress

Experiments of Schmid and Boas

- ◆ No influence of the normal stress operating on the glide plane.
- ◆ A critical value of the resolved shear stress is required for the initiation of glide.



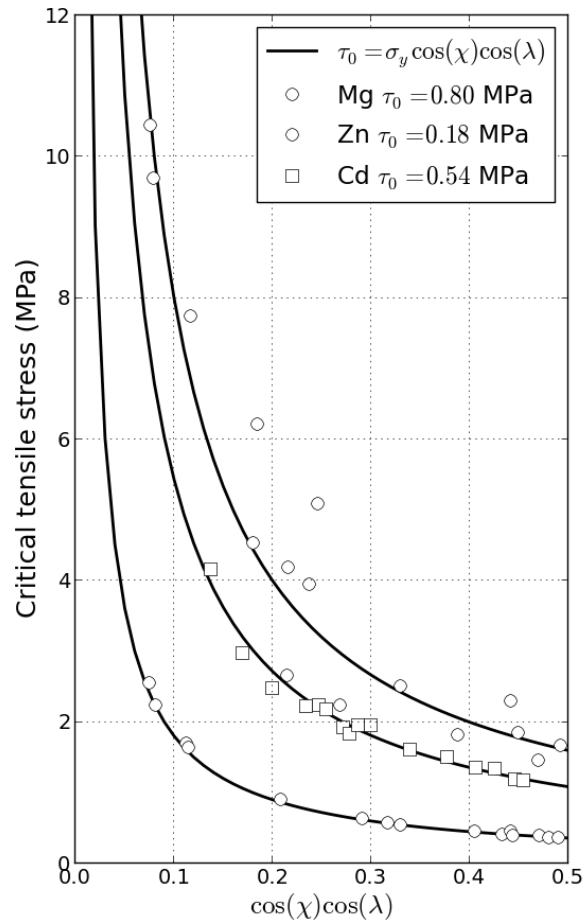
The critical resolved shear stress

More experimental data on critical resolved shear stress:

Mg data from [Schmid1950]

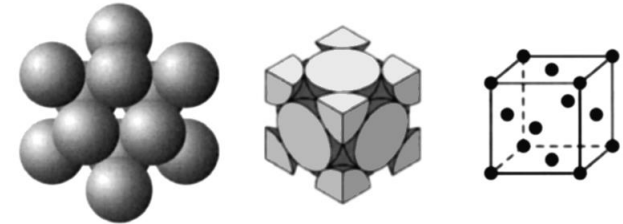
Zn data from [Jillson]

Cd data from [Andrade & Roscoe]



Some important crystalline structures

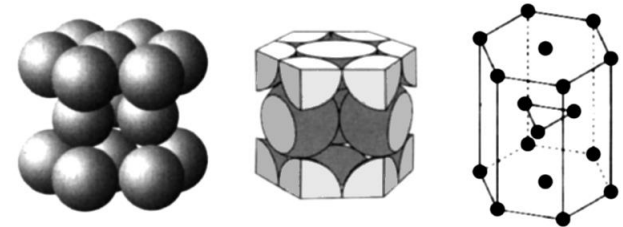
Face centered cubic



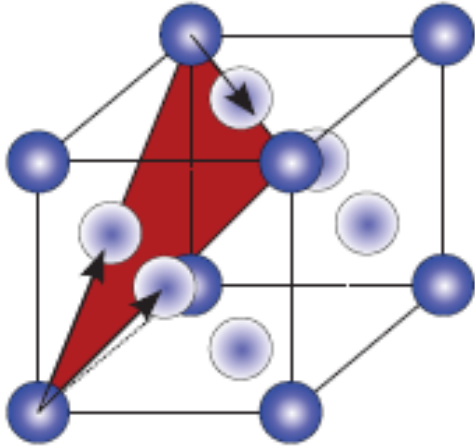
Body centered cubic



Hexagonal close packing



Slip systems in FCC crystals



Octahedral slip

4 slip planes $\{111\}$ i.e. (111) , $(\bar{1}\bar{1}1)$, $(11\bar{1})$, $(\bar{1}1\bar{1})$ indicated by normals

3 slip directions of type $\langle 110 \rangle$ in each 111 plane

number	1	2	3	4	5	6
name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	$(\bar{1}\bar{1}1)$	$(\bar{1}\bar{1}1)$	$(\bar{1}\bar{1}1)$
direction	$[\bar{1}01]$	$[0\bar{1}1]$	$[\bar{1}10]$	$[\bar{1}01]$	$[011]$	$[110]$
number	7	8	9	10	11	12
name	A2	A6	A3	C5	C3	C1
plane	$(\bar{1}11)$	$(\bar{1}11)$	$(\bar{1}11)$	$(11\bar{1})$	$(11\bar{1})$	$(11\bar{1})$
direction	$[0\bar{1}1]$	$[110]$	$[101]$	$[\bar{1}10]$	$[101]$	$[011]$

Simple tension for a single crystal specimen

Tensile test in the direction [001]

Schmid factor $m^s = (\mathbf{n}^s \cdot \mathbf{t}) (\mathbf{l}^s \cdot \mathbf{t})$

number	1	2	3	4	5	6
name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)
direction	$[\bar{1}01]$	$[0\bar{1}\bar{1}]$	$[\bar{1}\bar{1}0]$	$[\bar{1}01]$	$[011]$	$[110]$
Schmid factor						
number	7	8	9	10	11	12
name	A2	A6	A3	C5	C3	C1
plane	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	(111)	(111)	(111)
direction	$[0\bar{1}\bar{1}]$	$[110]$	$[101]$	$[\bar{1}\bar{1}0]$	$[101]$	$[011]$
Schmid factor						

Simple tension for a single crystal specimen

Tensile test in the direction [001]

8 active slip systems

Schmid factor $m^s = (\mathbf{n}^s \cdot \mathbf{t}) (\mathbf{l}^s \cdot \mathbf{t})$

number	1	2	3	4	5	6
name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	(1 $\bar{1}$ 1)	(1 $\bar{1}$ 1)	(1 $\bar{1}$ 1)
direction	[$\bar{1}$ 01]	[0 $\bar{1}$ 1]	[$\bar{1}$ 10]	[$\bar{1}$ 01]	[011]	[110]
Schmid factor	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	0	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	0
number	7	8	9	10	11	12
name	A2	A6	A3	C5	C3	C1
plane	($\bar{1}$ 11)	($\bar{1}$ 11)	($\bar{1}$ 11)	(11 $\bar{1}$)	(11 $\bar{1}$)	(11 $\bar{1}$)
direction	[0 $\bar{1}$ 1]	[110]	[101]	[$\bar{1}$ 10]	[101]	[011]
Schmid factor	$\frac{1}{\sqrt{6}}$	0	$\frac{1}{\sqrt{6}}$	0	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$

Simple tension for a single crystal specimen

Tensile test in the direction [111]

Schmid factor $m^s = (\mathbf{n}^s \cdot \mathbf{t}) (\mathbf{l}^s \cdot \mathbf{t})$

number	1	2	3	4	5	6
name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)
direction	$[\bar{1}01]$	$[0\bar{1}\bar{1}]$	$[\bar{1}\bar{1}0]$	$[\bar{1}01]$	$[011]$	$[110]$
Schmid factor						
number	7	8	9	10	11	12
name	A2	A6	A3	C5	C3	C1
plane	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	($11\bar{1}$)	($11\bar{1}$)	($11\bar{1}$)
direction	$[0\bar{1}\bar{1}]$	$[110]$	$[101]$	$[\bar{1}\bar{1}0]$	$[101]$	$[011]$
Schmid factor						

Simple tension for a single crystal specimen

Tensile test in the direction $[111]$

6 active slip systems

Schmid factor $m^s = (\mathbf{n}^s \cdot \mathbf{t}) (\mathbf{l}^s \cdot \mathbf{t})$

number	1	2	3	4	5	6
name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	$(1\bar{1}1)$	$(1\bar{1}1)$	$(1\bar{1}1)$
direction	$[\bar{1}01]$	$[0\bar{1}1]$	$[\bar{1}10]$	$[\bar{1}01]$	$[011]$	$[110]$
Schmid factor	0	0	0	0	$\frac{2}{3\sqrt{6}}$	$\frac{2}{3\sqrt{6}}$
number	7	8	9	10	11	12
name	A2	A6	A3	C5	C3	C1
plane	$(\bar{1}11)$	$(\bar{1}11)$	$(\bar{1}11)$	$(11\bar{1})$	$(11\bar{1})$	$(11\bar{1})$
direction	$[0\bar{1}1]$	$[110]$	$[101]$	$[\bar{1}10]$	$[101]$	$[011]$
Schmid factor	0	$\frac{2}{3\sqrt{6}}$	$\frac{2}{3\sqrt{6}}$	0	$\frac{2}{3\sqrt{6}}$	$\frac{2}{3\sqrt{6}}$

Simple tension for a single crystal specimen

Tensile test in the direction [011]

Schmid factor $m^s = (\mathbf{n}^s \cdot \mathbf{t}) (\mathbf{l}^s \cdot \mathbf{t})$

number	1	2	3	4	5	6
name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)
direction	$[\bar{1}01]$	$[0\bar{1}\bar{1}]$	$[\bar{1}\bar{1}0]$	$[\bar{1}01]$	$[011]$	$[110]$
Schmid factor						
number	7	8	9	10	11	12
name	A2	A6	A3	C5	C3	C1
plane	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	($\bar{1}\bar{1}\bar{1}$)	(111)	(111)	(111)
direction	$[0\bar{1}\bar{1}]$	$[110]$	$[101]$	$[\bar{1}\bar{1}0]$	$[101]$	$[011]$
Schmid factor						

Simple tension for a single crystal specimen

Tensile test in the direction $[011]$

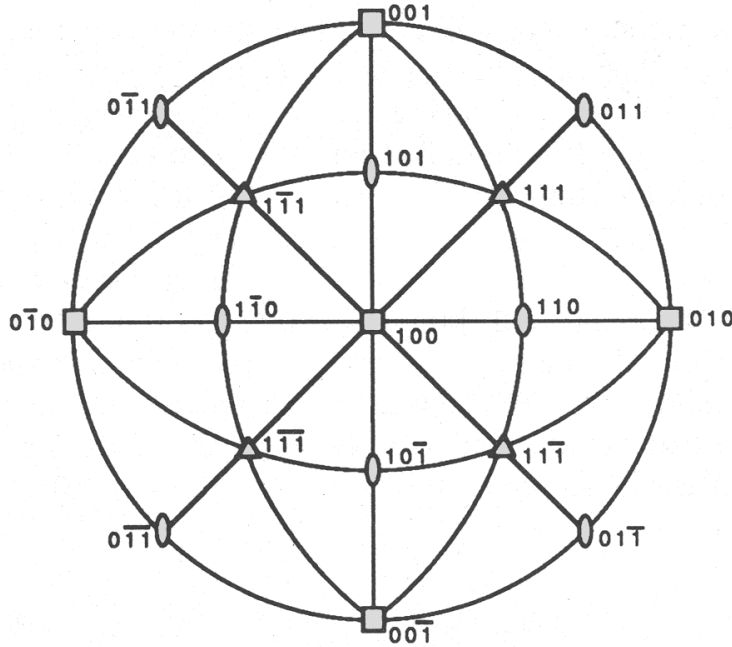
4 active slip systems

Schmid factor $m^s = (\mathbf{n}^s \cdot \mathbf{t}) (\mathbf{l}^s \cdot \mathbf{t})$

number	1	2	3	4	5	6
name	B4	B2	B5	D4	D1	D6
plane	(111)	(111)	(111)	$(\bar{1}\bar{1}1)$	$(1\bar{1}\bar{1})$	$(\bar{1}\bar{1}\bar{1})$
direction	$[\bar{1}01]$	$[0\bar{1}1]$	$[\bar{1}10]$	$[\bar{1}01]$	$[011]$	$[110]$
Schmid factor	$\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	0	0	0
number	7	8	9	10	11	12
name	A2	A6	A3	C5	C3	C1
plane	$(\bar{1}11)$	$(\bar{1}11)$	$(\bar{1}11)$	$(11\bar{1})$	$(11\bar{1})$	$(11\bar{1})$
direction	$[0\bar{1}1]$	$[110]$	$[101]$	$[\bar{1}10]$	$[101]$	$[011]$
Schmid factor	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	0	0	0

Summary: inverse pole figure

First activated slip systems in tension according to Schmid law:



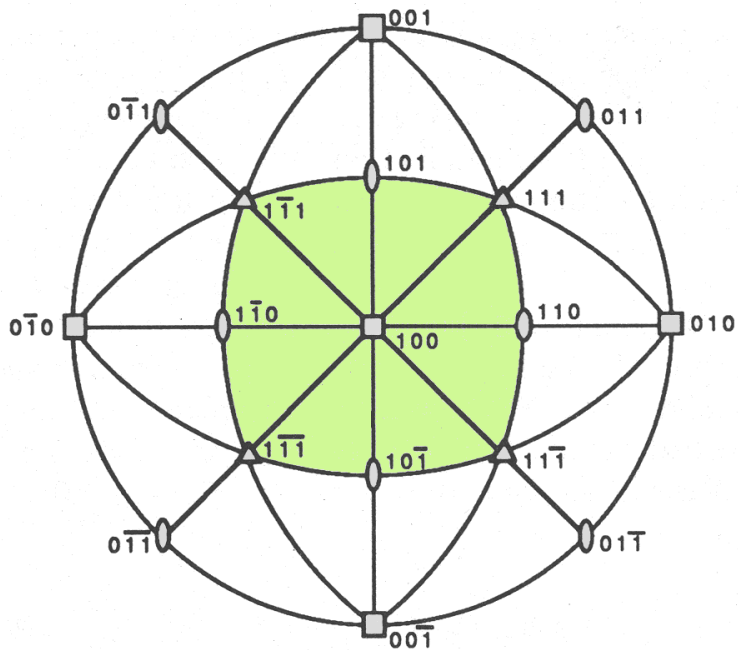
[001] tension

[111] tension

[110] tension

Summary: inverse pole figure

First activated slip systems in tension according to Schmid law:



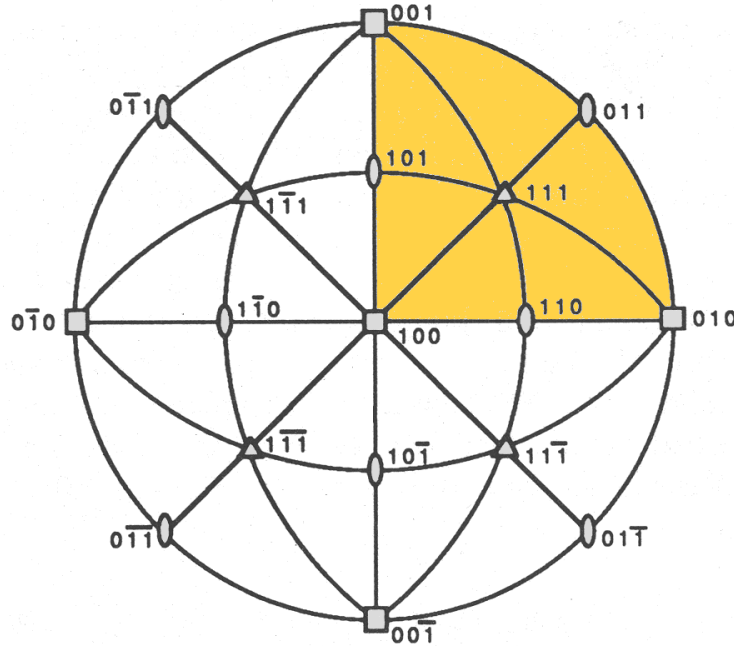
[001] tension
8 slip systems :
B4, C1, A2, D4,
C3, B2, D1, A3

[111] tension

[110] tension

Summary: inverse pole figure

First activated slip systems in tension according to Schmid law:



[001] tension

8 slip systems :
B4, C1, A2, D4,
C3, B2, D1, A3

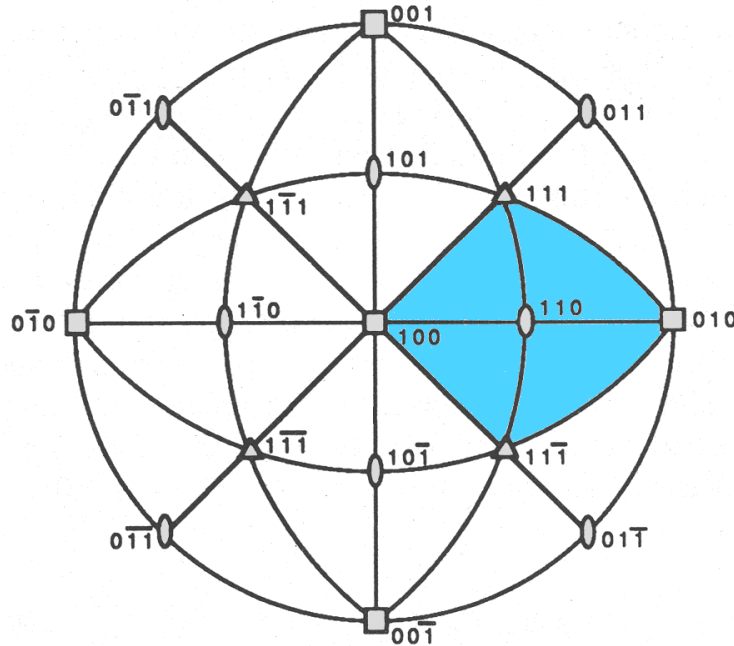
[111] tension

6 slip systems :
D4, D1, C1, C5,
B4, B5

[110] tension

Summary: inverse pole figure

First activated slip systems in tension according to Schmid law:



[001] tension

8 slip systems :
B4, C1, A2, D4,
C3, B2, D1, A3

[111] tension

6 slip systems :
D4, D1, C1, C5,
B4, B5

[110] tension

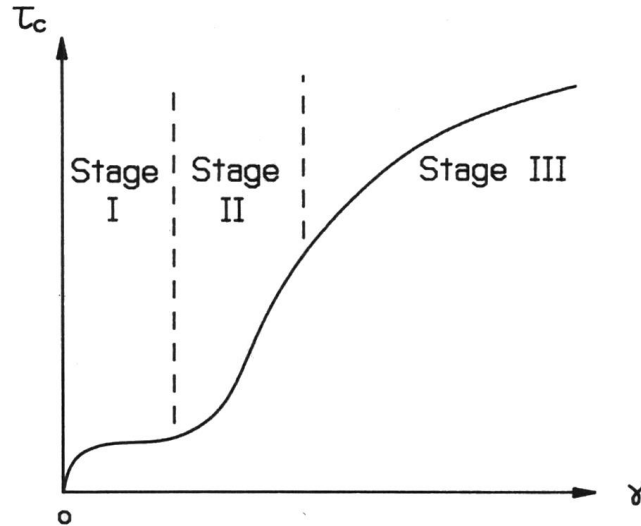
4 slip systems :
B4, A3, B5, A6

Historical developments of the theory

- ❑ The initial foundations are laid out by the pioneering work of **Taylor** in 1938 [Taylor, 1938].
- ❑ The theory was since further developed by **Bishop & Hill** [Bishop and Hill, 1951], **Kocks** [Kocks and Brown, 1966, Kocks, 1970], **Hill and Rice** [Hill, 1966, Hill and Rice, 1972], **Asaro and Rice** [Asaro, 1975, Asaro and Rice, 1977], **Havner** [Havner, 1992], **Cailletaud** [Meric et al., 1991, Meric and Cailletaud, 1991].

Rate sensitive plastic slip

The Schmid law predicting an elastic-perfectly plastic behaviour cannot describe typical (τ_c) curves. The plastic slip on a given slip system s (τ^s , $\dot{\gamma}^s$) is rate sensitive and can be described by a power law.



Two main formulations exist in the literature :

Multiplicative

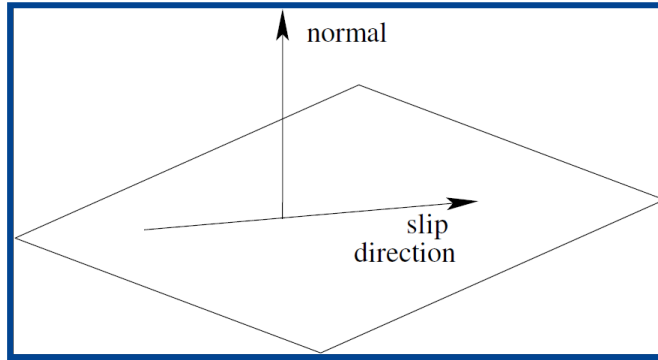
$$\dot{\gamma}^s = \dot{\gamma}_0 \left(\frac{\tau^s}{\tau_0} \right)^n \text{sign}(\tau^s)$$

Additive with threshold

$$\dot{\gamma}^s = \left\langle \frac{|\tau^s| - \tau_0}{K} \right\rangle \text{sign}(\tau^s)$$

τ_0 is the critical resolved shear stress, $\dot{\gamma}_0$, K and n are parameters.

Basic ingredients of crystal plasticity



A collection of N slip systems

Normal to slip plane \vec{n}^s

Direction of slip system \vec{l}^s

Orientation tensor

$$\underline{\underline{m}}^s = \frac{1}{2}(\vec{n}^s \otimes \vec{l}^s + \vec{l}^s \otimes \vec{n}^s)$$

Resolved shear stress

$$\tau^s = \underline{\underline{\sigma}} : \underline{\underline{m}}^s = \underline{\underline{\sigma}} : \frac{1}{2}(\vec{l}^s \otimes \vec{n}^s + \vec{n}^s \otimes \vec{l}^s)$$

Yield function

$$f^s(\tau^s, \text{hardening variables}, \dots) = 0$$

Elastic behavior

$$\forall s, f^s < 0$$

Shear strain rate

$$\dot{\gamma}^s = \dot{\gamma}^s(f^s)$$

Small strain formalism and schmid law

- Strain partition into elastic and plastic parts

$$\underline{\varepsilon} = \underline{\varepsilon}^e + \underline{\varepsilon}^p$$

- Resolved shear stress, computed by means of the *orientation tensor*, \underline{m}^s , using the vector normal to the slip plane, \underline{n}^s , and the slip direction, \underline{l}^s

$$\tau^s = \underline{\sigma} : \underline{m}^s \quad \underline{m}^s = \frac{1}{2}(\bar{l}^s \otimes \bar{n}^s + \bar{n}^s \otimes \bar{l}^s)$$

- A yield function f^s is defined on each slip system s (here with additive hardening)

$$f^s = |\tau^s - x^s| - r^s$$

- Viscoplastic flow Shear strain rate s deduced from resolved shear stress s and from the value of the kinematic (Xs) and isotropic (Rs) variables :

$$\dot{\gamma}^s = \left\langle \frac{|\tau^s - x^s| - r^s}{K^s} \right\rangle^{n^s} \text{sign}(\tau^s - x^s)$$

Single crystal model

- Viscoplastic potential [Mandel, 1972]

$$\Omega(\underline{\sigma}, \dots) = \sum_s \frac{K}{n+1} \left\langle \frac{f^s}{K} \right\rangle^{n+1}$$

- Viscoplastic strain rate (from the viscoplastic potential)

$$\underline{\dot{\epsilon}}^p = \sum_s \frac{\partial \Omega}{\partial \underline{\sigma}} = \sum_s \frac{\partial \Omega}{\partial f^s} \frac{\partial f^s}{\partial \underline{\sigma}} = \sum_s \dot{v}^s \underline{m}^s \eta^s = \sum_s \dot{\gamma}^s \underline{m}^s$$

- Hardening rules : X^s and R^s computed by means of state variables s and r^s

$$\begin{aligned} X^s &= c \alpha^s & \dot{\alpha}^s &= (\eta^s - d \alpha^s) \dot{v}^s \\ R^s &= r_0 + Q \sum_j h_{sj} r^j & \dot{r}^j &= (1 - b r^j) \dot{v}^j \end{aligned}$$

Interaction matrix

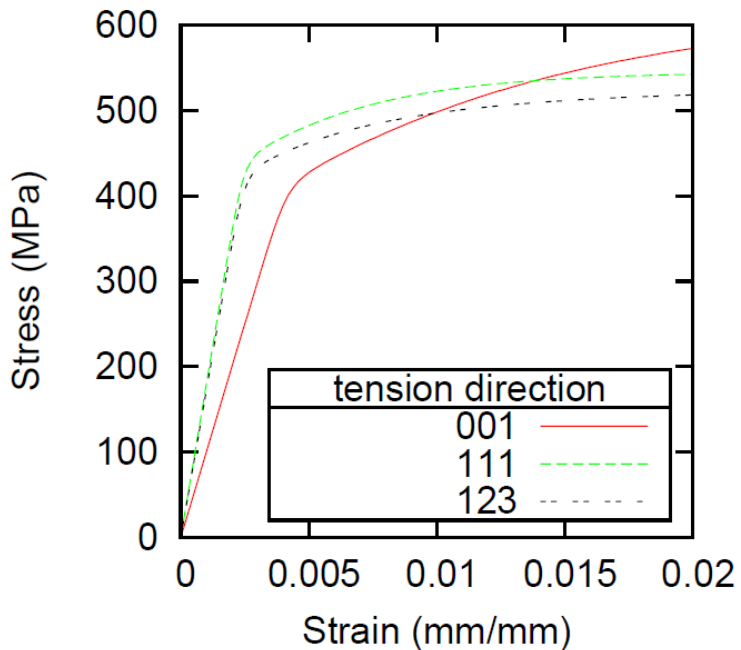
	B4	B2	B5	D4	D1	D6	A2	A6	A3	C5	C3	C1
B4	h_1	h_2	h_2	h_4	h_5	h_5	h_5	h_6	h_3	h_5	h_3	h_6
B2		h_1	h_2	h_5	h_3	h_6	h_4	h_5	h_5	h_5	h_6	h_3
B5			h_1	h_5	h_6	h_3	h_5	h_3	h_6	h_4	h_5	h_5
D4				h_1	h_2	h_2	h_6	h_5	h_3	h_6	h_3	h_5
D1					h_1	h_2	h_3	h_5	h_6	h_5	h_5	h_4
D6						h_1	h_5	h_4	h_5	h_3	h_6	h_5
A2							h_1	h_2	h_2	h_6	h_5	h_3
A6								h_1	h_2	h_3	h_5	h_6
A3									h_1	h_5	h_4	h_5
C5										h_1	h_2	h_2
C3											h_1	h_2
C1												h_1

simplified version (Taylor model)

$$h_1 = h_2 = h_3 = h_4 = h_5 = h_6 = 1$$

Example of monocrystal behaviour

Stress–strain curves simulated for a β -titane monocrystal under tension in different directions.



Cubic elasticity

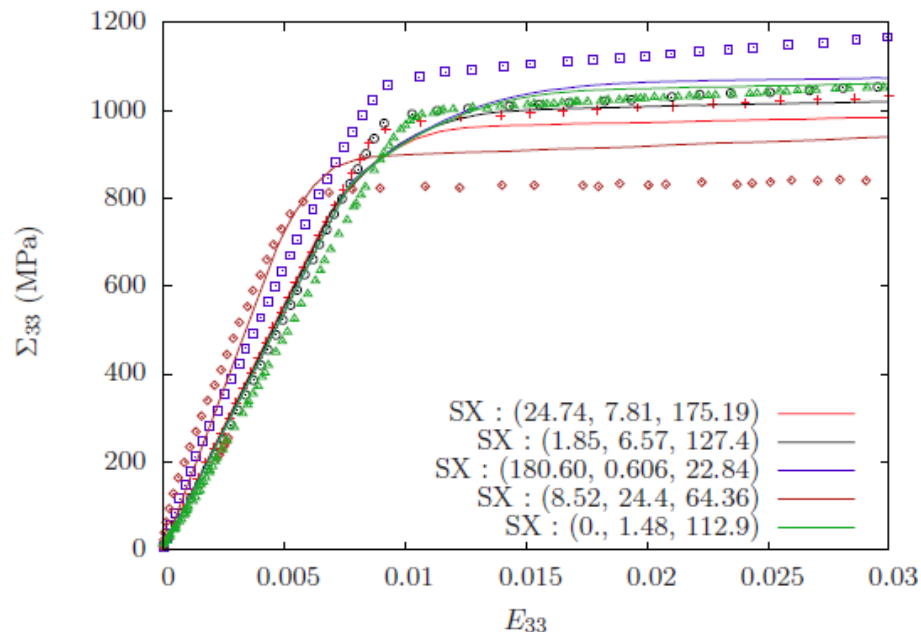
C11 (MPa)	150000
C12 (MPa)	75000
C44 (MPa)	112000

Crystal plasticity

τ_0 (MPa)	Q (MPa)	b	K (Mpa)	n	h_{ij}
100	10	350	500	5	1

Example of monocrystal behaviour

Stress–strain curves simulated for a nickel bas superalloys monocrystal under tension in different directions at 650 °C.



Temperature : 650 °C

$$C_{1111} = 203760 \text{ MPa}$$

$$C_{1122} = 125490 \text{ MPa}$$

$$C_{1212} = 112000 \text{ MPa}$$

« fast » potential

$$n = 11.$$

$$K = 10.3 \text{ MPa} \cdot \text{s}^{\frac{1}{n}}$$

$$\tau_c = 337 \text{ MPa}$$

$$c1 = 110\,425 \text{ MPa}$$

$$d1 = 1\,165$$

$$c2 = 504 \text{ MPa}$$

« slow » potential

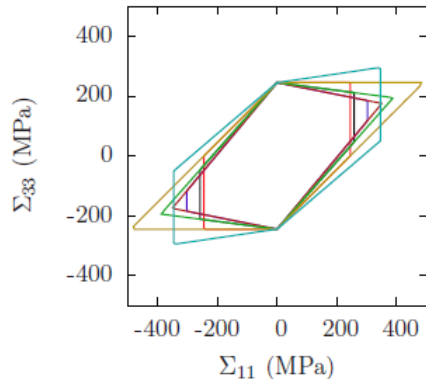
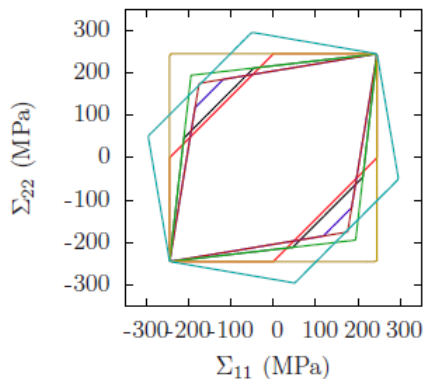
$$n = 6.5$$

$$K = 2000 \text{ MPa} \cdot \text{s}^{\frac{1}{n}}$$

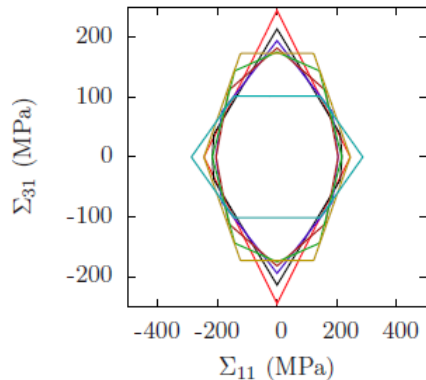
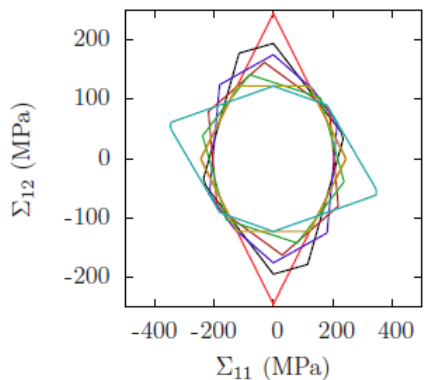
$$\tau_c = 1000 \text{ MPa}$$

CONFIDENTIEL / DATE / DIRECTION

Yield surface



Monocrystal $\langle 001 \rangle$ ———
 $\phi_1 = 9^\circ$ ———
 $\phi_1 = 18^\circ$ ———
 $\phi_1 = 27^\circ$ ———
 $\phi_1 = 36^\circ$ ———
 Monocrystal $\langle 011 \rangle$ ———
 Monocrystal $\langle 111 \rangle$ ———



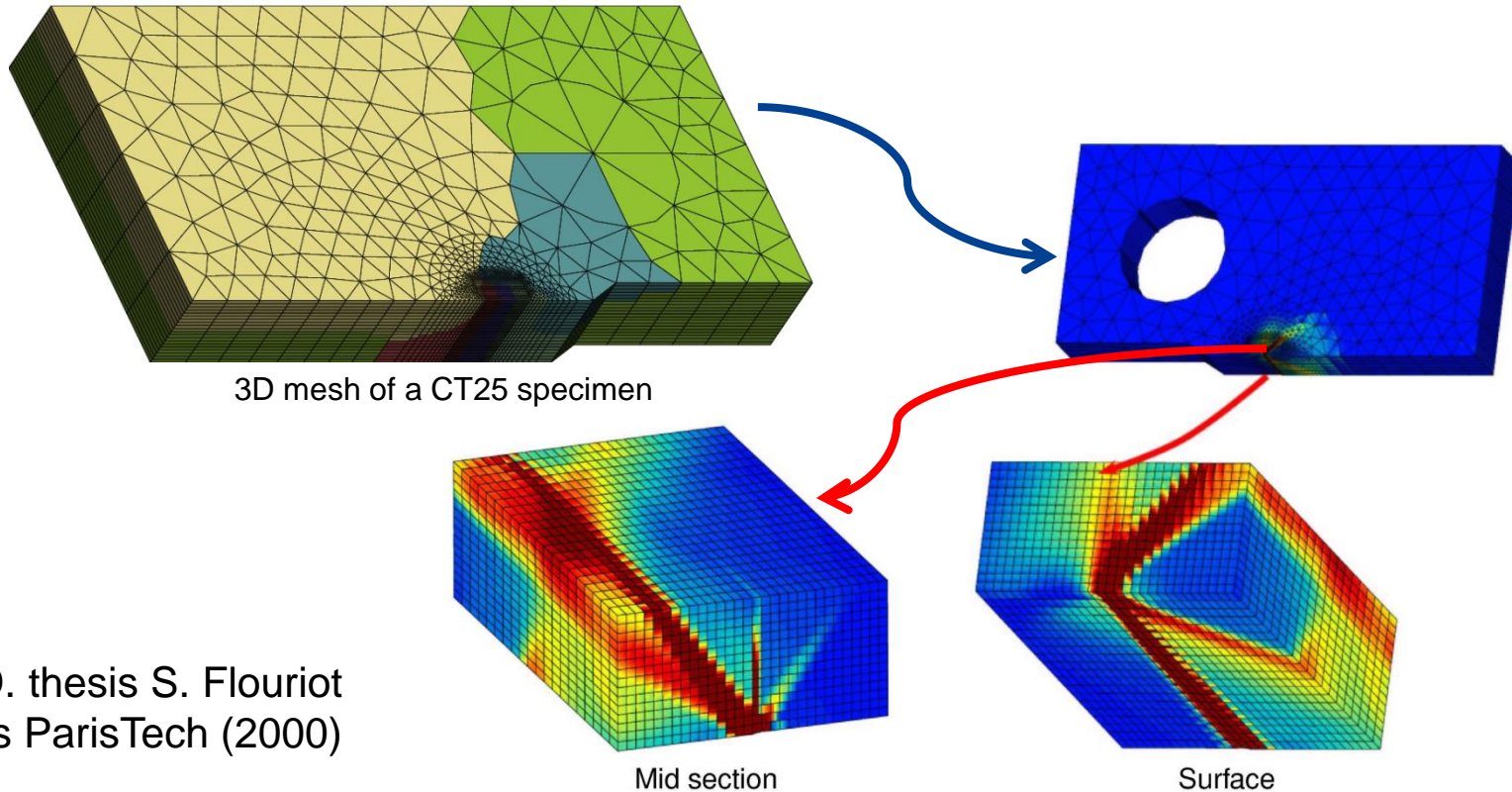
Monocrystal $\langle 001 \rangle$ ———
 $\phi_1 = 9^\circ$ ———
 $\phi_1 = 18^\circ$ ———
 $\phi_1 = 27^\circ$ ———
 $\phi_1 = 36^\circ$ ———
 Monocrystal $\langle 011 \rangle$ ———
 Monocrystal $\langle 111 \rangle$ ———

Z-sim input file to compute the yield surface

```
****simulate
***test polysurf
**load
*segment 1
  time sig11 sig22 sig33 sig12 sig23 sig31
    0.0 0. 0. 0. 0. 0. 0.
    0.01 0. 1. 0. 0. 0. 0.
**model
*file ti.mat
*rotation euler x1 1. 0. 0. x2 1. 1. 1.
*integration runge_kutta 1.e-4
**output
**yield_surface ps-11-12.test
*degrees 1.
*factor 1.e9
*find_offset 1.e-1
*component sig11 sig12
*time 0.0
****return
```



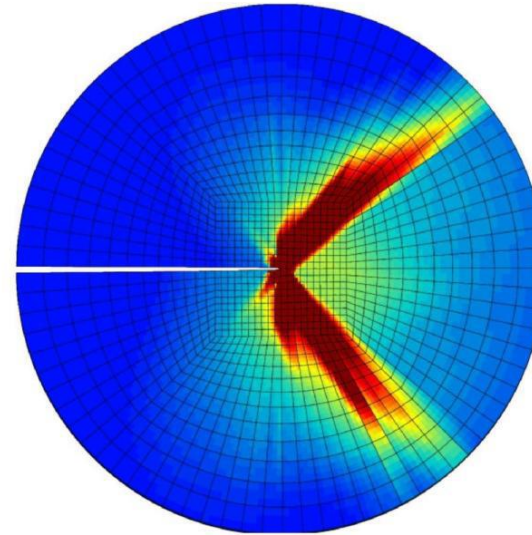
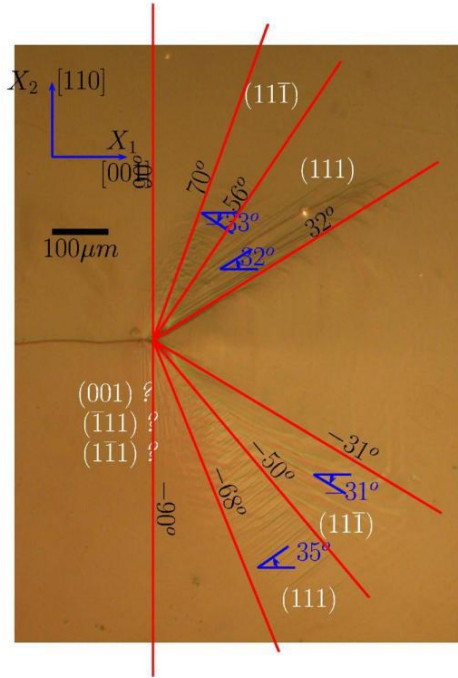
Crack in Ni based superalloy single crystals



Ph. D. thesis S. Flouriot
Mines ParisTech (2000)

Crack in Ni based superalloy single crystals

Comparison with experiments [Flouriot et al., 2003]



Surface of the specimen
3D F.E. Simulation

Small strain crystal plasticity – AM1

Cubic elasticity $\underline{\sigma} = \underline{\underline{C}} : \underline{\varepsilon}^e$

Resolved shear stress $\tau^s = \underline{\sigma} : \underline{m}^s$

Viscous flow $\dot{\gamma}^s = \varepsilon_0^s \sinh \left(\left\langle \frac{|\tau^s - x^s| - r^s}{K^s} \right\rangle^{n^s} \right) \text{sign}(\tau^s - x^s)$

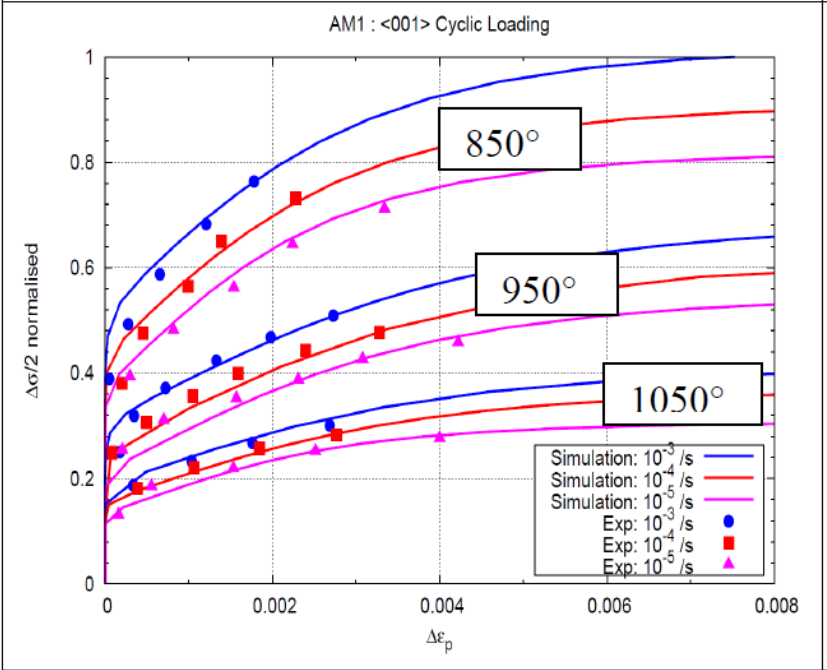
Plastic strain rate $\underline{\dot{\varepsilon}}^p = \sum_s \dot{\gamma}^s \underline{m}^s$

Orientation tensor $\underline{\underline{m}}^s = \frac{1}{2} (\underline{\bar{n}}^s \otimes \underline{\bar{l}}^s + \underline{\bar{l}}^s \otimes \underline{\bar{n}}^s)$

Kinematic hardening restoration $\dot{x}^s = C^s \dot{\gamma} - D^s x^s |\dot{\gamma}^s| - \left(\frac{|x^s|}{M^s} \right)^{m^s} \text{sign}(x^s)$

Small strain crystal plasticity – AM1

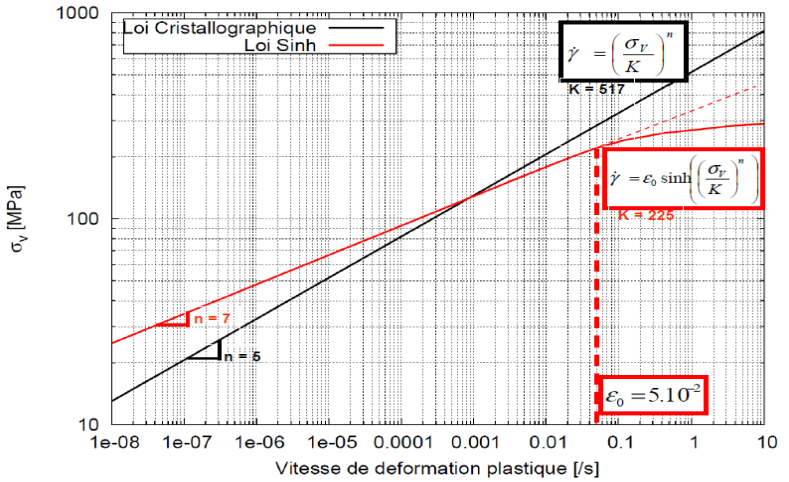
saturation of viscosity



Comparison between simulations and experiments for some isothermal cyclic strain tests.

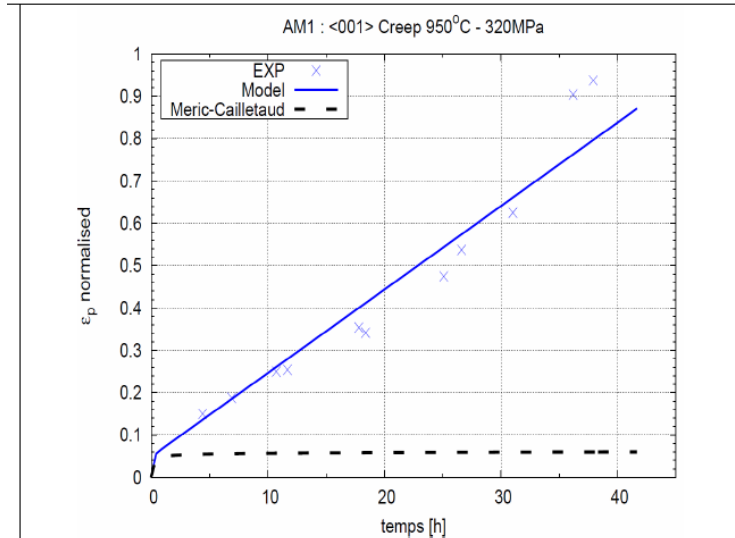
Use of sinh instead of power law (Norton-Hoff)

Goals : the use of a “sinh” form limits the increase of the viscous stress when the strain rate increases

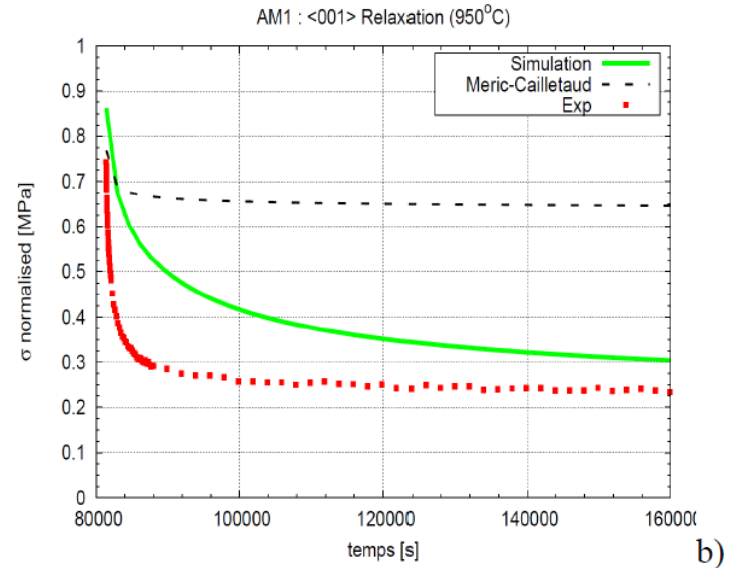


Small strain crystal plasticity – AM1

Concerning creep tests, the introduction of a static recovery term enables to simulate with quite good accuracy the secondary creep



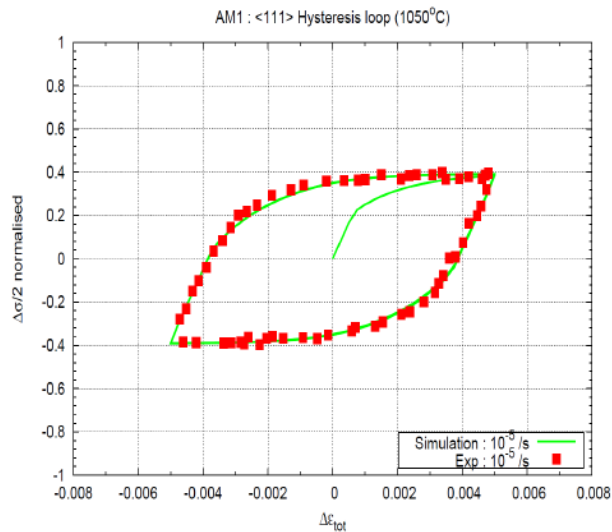
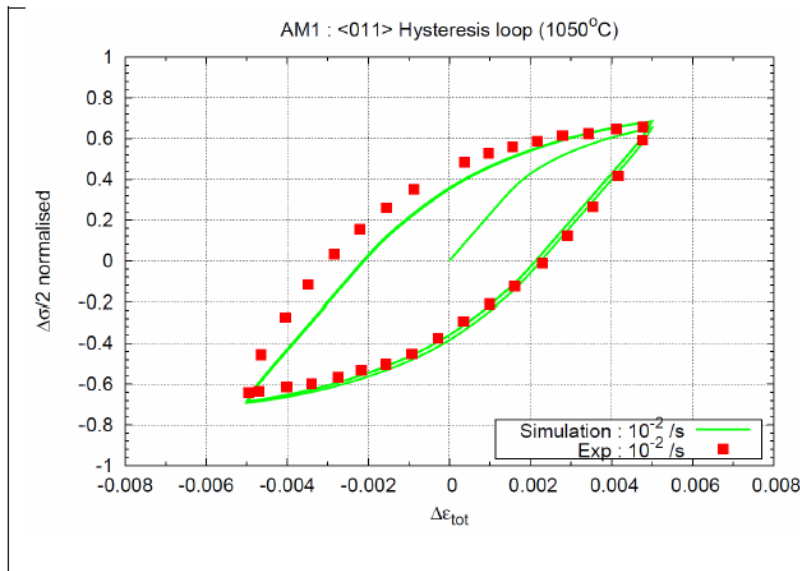
Comparison between simulations and experiments for some creep tests.



Long term relaxation at 950°C

Small strain crystal plasticity – AM1

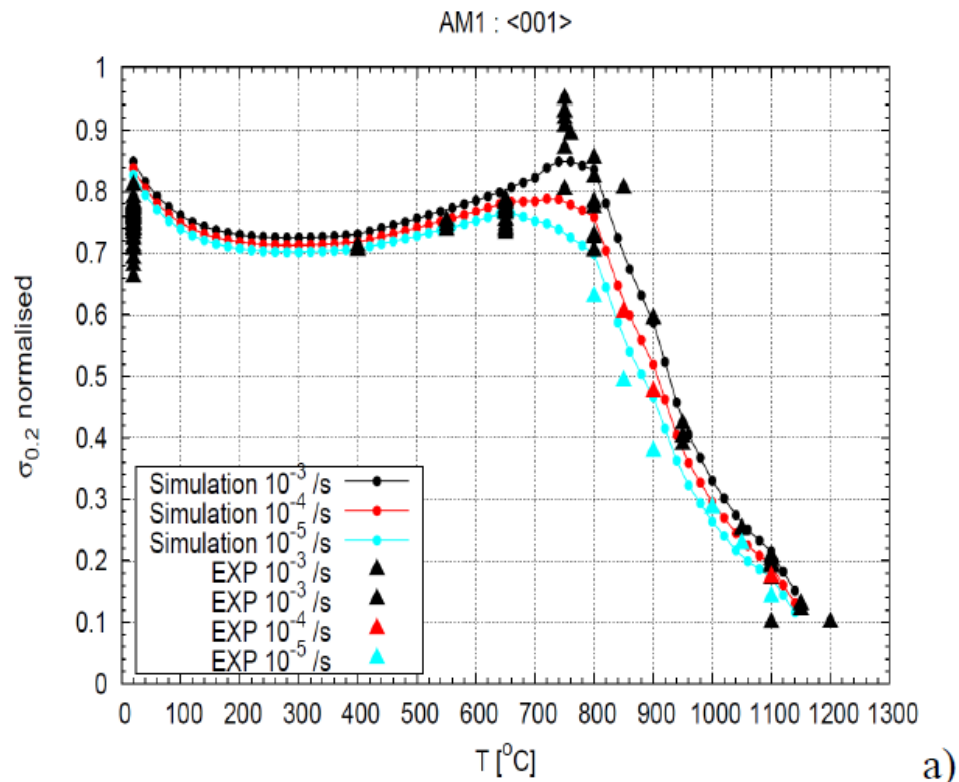
Hysteretic loop at 950 °C



Comparison between simulations and experiments for some hysteresis loops performed along <011> and <111> crystallographic directions

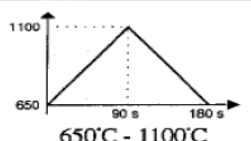
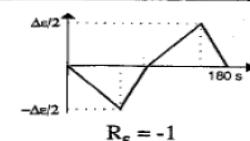
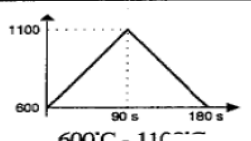
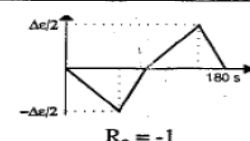
Small strain crystal plasticity – AM1

Simulation of the evolution of the 0,2% yield stress with temperature for three strain rates in the <001> direction



Small strain crystal plasticity – AM1

In a second step, in order to test the material under conditions closer than those observed in service, specific tests were developed called: thermomechanical tests. In such tests, the material is simultaneously subjected to mechanical strain and temperature, both, evolving during the cycle.

Dénomination du cycle	Cyclage en température	Cyclage mécanique
S ₁	 <p>650°C - 1100°C</p>	 <p>$R_{\epsilon} = -1$</p>
S ₂	 <p>600°C - 1100°C</p>	 <p>$R_{\epsilon} = -1$</p>

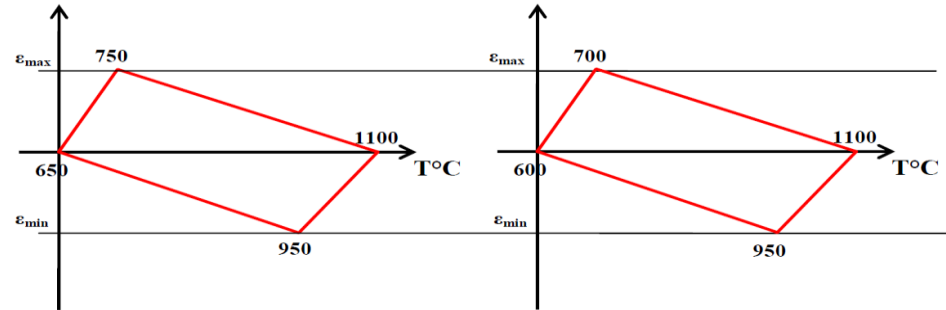
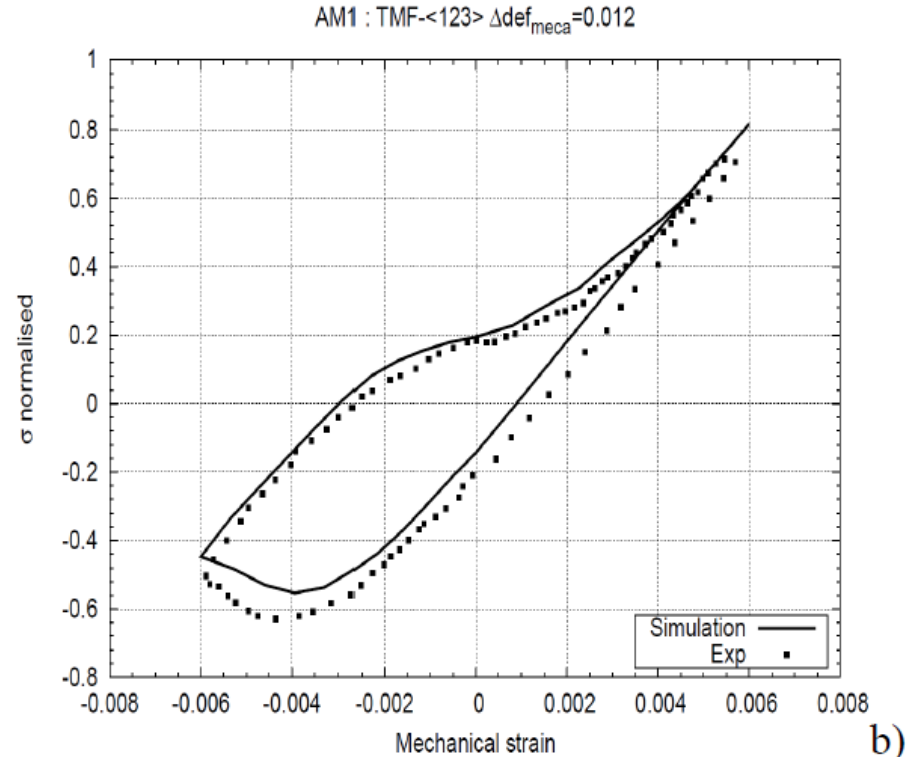


Figure 54.- Évolution de la déformation mécanique en fonction de la température : a) cycle S₁, b) cycle S₂.

Small strain crystal plasticity – AM1

Comparison between simulations and experiments for some anisothermal test



Small strain crystal plasticity – AM1

Synthèse avantage/inconvénient des différents modèles de comportement

	Modèle isotrope à 2 Potentiels de Norton	Modèle anisotrope cristallographique initial à 1 Potentiel de Norton	Modèle « Sinh+restauration »
Anisotrope cristalline	NON	OUI	OUI
Prise en compte de la restauration statique de l'écroutissage	NON	NON	OUI
Prise en compte du fluage	OUI <i>mais <u>uniquement</u> dans la direction <001> du monocristal</i>	NON	OUI
Saturation de la viscosité	NON	NON	OUI

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